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Low-energy moments of non-diagonal quark current correlators at four loops

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Abstract

We compute the leading four physical terms in the low-energy expansions of heavy–light quark current correlators at four-loop order. As a by-product we reproduce the corresponding top-induced non-singlet correction to the electroweak ρ parameter.

1. Introduction

Two-point correlation functions of heavy–light quark currents have found use in a number of phenomenological applications. One example is the prediction of corrections to the electroweak ρ parameter [1–3], where the flavour non-diagonal correlator of vector currents is required for vanishing external momentum. Another important class of applications is the sum-rule determination of meson decay constants (see e.g. [4,5]). Here, the absorptive part of the respective correlators above the production threshold is needed.

Progress in lattice simulation may allow precise determinations of even more QCD parameters. For instance, the values of the strong coupling constant, the charm quark mass and the bottom quark mass have been determined with high accuracy from moments of heavy–heavy

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correlators in [6,7]. In these analyses, moments of flavour diagonal currents have been determined on the lattice choosing a frame where the spatial momentum of the correlators vanishes. The values of the quark masses and the coupling constant are then extracted by equating these moments to their counterparts calculated in perturbation theory at the four-loop order [8–14].

The methodology is thus similar to traditional quarkonium sum rules [15–17], but using lattice moments in place of moments of the experimentally measured hadronic R ratio. While for the sum rules only the correlator of vector currents can be used, there is no such restriction for the lattice simulation. In fact, in [6] different Lorentz structures were considered, with the most precise results stemming from pseudo-scalar currents. Furthermore, also correlators of heavy–light currents could be used to extract the values of the charm and bottom quark masses and possibly the strong coupling constant [18]. To be competitive with the analyses for the heavy–heavy case the corrections to the perturbative moments of the heavy–light current correlators have to be known up to four loops. These corrections are presented in this work.

Given their usefulness, perturbative corrections to heavy–light correlators have been studied quite intensively and analytic results up to two loops have been known for many years [19,20]. While the three-loop correction is not known analytically, many terms in expansions in both the low-energy and the high-energy limit have been calculated in [21–23]. Combining these with the behaviour near threshold, accurate approximations for arbitrary kinematics have been constructed [21,22]. In the low-energy region also corrections due to a non-vanishing light quark mass are known [24,25].

The four-loop corrections remain mostly unknown. In the high-energy region the leading term is equal to the non-singlet part of the corresponding diagonal correlator, which has been computed for both scalar and vector currents [26–28]. In the low-energy region, conversely, there is no such simple correspondence between diagonal and non-diagonal correlators. The vector correlator in the limit of vanishing external momentum constitutes a central ingredient in the determination of non-singlet four-loop corrections to the ρ parameter, which have been calculated in [2,3].

In this work we present the four-loop corrections to the low-energy expansions of both scalar and vector heavy–light quark current correlators up to the eighth power of the external momentum. After introducing our conventions in Section 2, we briefly describe the calculational setup and present our results in Section 3. Section 4 describes the re-calculation of the top-induced contributions to the electroweak ρ parameter, which constitutes an important consistency check. We conclude in Section 5.

2. Conventions

The correlators of heavy–light vector and scalar currents are defined as

$$\Pi_{\mu\nu}(q) = i\int dx\, e^{iqx}\langle 0|j_\mu(x)j_\nu(0)|0\rangle\,, \tag{1}$$

$$\Pi(q) = i\int dx\, e^{iqx}\langle 0|j(x)j(0)|0\rangle \tag{2}$$

with the vector current $j_\mu(x) = \bar\psi(x)\gamma_\mu\chi(0)$ and the scalar current $j(x) = \bar\psi(x)\chi(0)$. We consider a heavy quark ψ with the pole mass m and a massless light quark χ. It should be noted that in the limit of a vanishing light-quark mass the correlators of two axial-vector or pseudo-scalar currents coincide with the vector and scalar correlators, respectively.

It is convenient to introduce polarisation functions