Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 899 (2015) 513–526
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Scattering of nucleons in the classical Skyrme model

David Foster [a,*], Nicholas S. Manton [b,*]

[a] *School of Physics, HH Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, UK*
[b] *Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK*

Received 2 July 2015; accepted 13 August 2015

Available online 18 August 2015

Editor: Stephan Stieberger

Abstract

Classically spinning $B = 1$ Skyrmions can be regarded as approximations to nucleons with quantised spin. Here, we investigate nucleon–nucleon scattering through numerical collisions of spinning Skyrmions. We identify the dineutron/diproton and dibaryon short-lived resonance states, and also the stable deuteron state. Our simulations lead to predictions for the polarisation states occurring in right angle scattering.

1. Introduction

The Skyrme model [1] is a nonlinear theory of pion fields that has topological soliton solutions. Encouragingly, Witten identified the Skyrme model as a low energy effective model of QCD [2,3]. The conserved topological charge is interpreted as the baryon number B, and the minimal energy static solutions for each integer B are called Skyrmions. They can be treated as rigid bodies, free to rotate in space and also in isospace (the three-dimensional space of the pion fields). When the rotational motion is quantised, the Skyrmions are models for nucleons and nuclei. In particular the $B = 1$ Skyrmions, quantised with spin and isospin half, model protons and neutrons.

* Corresponding authors.
E-mail addresses: dave.foster@bristol.ac.uk (D. Foster), N.S.Manton@damtp.cam.ac.uk (N.S. Manton).

http://dx.doi.org/10.1016/j.nuclphysb.2015.08.012
0550-3213/

A full quantum mechanical treatment of the interaction of two $B = 1$ Skyrmions is difficult. Each Skyrmion has three position coordinates and three orientational coordinates, so two-Skyrmion dynamics involves twelve coordinates [4]. A truncation to ten coordinates has been useful for modelling the deuteron bound state [5], but no proper discussion of two-nucleon scattering is possible with this truncation.

An alternative is a classical approach, using the idea that a classically spinning Skyrmion is a reasonable model of a nucleon [6,7]. The classical angular velocity of the Skyrmion is fixed to match the quantised spin and isospin of the nucleon. Scattering of non-spinning Skyrmions was first performed using an axially symmetric ansatz [8], the first full-field simulation was reported in [9], and multi-charge Skyrmion scattering was considered in [10]. The scattering of spinning Skyrmions representing nucleons was considered by Gisiger and Paranjape [6], and they obtained analytical formulae for the scattering angle for large impact parameters. There has also been some recent work on multi-charge, purely isospinning Skyrmions [12].

A systematic, numerical investigation of Skyrmion scattering without spin, at moderate and small impact parameters [11], confirmed that matter is exchanged between the Skyrmions when the Skyrmions are close, and also showed that a substantial rotation of the Skyrmion's orientation can occur. Here, we numerically investigate the scattering of two classically spinning $B = 1$ Skyrmions to model proton–proton, neutron–neutron and proton–neutron scattering at various impact parameters, including head-on collisions. We are particularly interested in the change in the polarisation states of the nucleons when they scatter, and also in finding evidence, using our classical approximation, for the deuteron and for the known two-nucleon resonance states.

The format of this paper is as follows. We first introduce the Skyrme model and its calibration. Then we discuss how to identify classically spinning Skyrmions as protons or neutrons. The next section presents the results of our numerical scattering of Skyrmions, and our identification of the dineutron/diproton resonances, the deuteron bound state, and the excited dibaryon state. The concluding section summarises and discusses our results.

2. The Skyrme model

The Skyrme model is defined by the Lagrangian density

$$\mathcal{L} = -\frac{F_\pi^2}{16}\,\mathrm{Tr}(R_\mu R^\mu) + \frac{1}{32e^2}\,\mathrm{Tr}([R_\mu, R_\nu][R^\mu, R^\nu]) + \frac{m_\pi^2 F_\pi^2}{8}\,\mathrm{Tr}(U - I_2)\,, \tag{1}$$

where the Skyrme field $U(t, \boldsymbol{x})$ is an SU(2)-valued scalar and $R_\mu = \partial_\mu U U^\dagger$ is its su(2)-valued current. F_π, e and m_π are parameters, which are fixed by comparison with experimental data. Their values will be discussed later. It is convenient for us to work in dimensionless Skyrme units. One Skyrme length unit corresponds to $\frac{2}{eF_\pi}$ in inverse MeV, and one Skyrme energy unit corresponds to $\frac{F_\pi}{4e}$ MeV. Conversion of inverse MeV to fm is as usual through $\hbar = 197.3$ MeV fm. The dimensionless pion mass in Skyrme units is

$$m = \left(\frac{2}{eF_\pi}\right) m_\pi\,. \tag{2}$$

In Skyrme units, the energy of a static field is

$$E = \int \left(-\frac{1}{2}\,\mathrm{Tr}(R_i R_i) - \frac{1}{16}\,\mathrm{Tr}([R_i, R_j][R_i, R_j]) + m^2\,\mathrm{Tr}(I_2 - U) \right) d^3x\,.$$