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Boundary algebras and Kac modules for logarithmic minimal models

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Abstract

Virasoro Kac modules were originally introduced indirectly as representations whose characters arise in the continuum scaling limits of certain transfer matrices in logarithmic minimal models, described using Temperley–Lieb algebras. The lattice transfer operators include seams on the boundary that use Wenzl–Jones projectors. If the projectors are singular, the original prescription is to select a subspace of the Temperley–Lieb modules on which the action of the transfer operators is non-singular. However, this prescription does not, in general, yield representations of the Temperley–Lieb algebras and the Virasoro Kac modules have remained largely unidentified. Here, we introduce the appropriate algebraic framework for the lattice analysis as a quotient of the one-boundary Temperley–Lieb algebra. The corresponding standard modules are introduced and examined using invariant bilinear forms and their Gram determinants. The structures of the Virasoro Kac modules are inferred from these results and are found to be given by finitely generated submodules of Feigin–Fuchs modules. Additional evidence for this identification is obtained by comparing the formalism of lattice fusion with the fusion rules of the Virasoro Kac modules. These are obtained, at the character level, in complete generality by applying a Verlinde-like formula and, at the module level, in many explicit examples by applying the Nahm–Gaberdiel–Kausch fusion algorithm.

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1. Introduction

The minimal models introduced by Belavin, Polyakov and Zamolodchikov [1] are central to conformal field theory [2]. A minimal model is characterised by a pair of co-prime integers, $1 < p < p'$, and is often denoted accordingly by $\mathcal{M}(p, p')$. The corresponding central charge c and conformal weights $\Delta_{r,s}$ are given by

$$c = 1 - 6\frac{(p'-p)^2}{pp'}, \qquad \Delta_{r,s} = \frac{(rp'-sp)^2 - (p'-p)^2}{4pp'}, \tag{1.1}$$

where $r = 1, 2, \ldots, p-1$ and $s = 1, 2, \ldots, p'-1$. These weights satisfy $\Delta_{r,s} = \Delta_{p-r,p'-s}$ and there is an irreducible Virasoro representation associated with each distinct conformal weight. Moreover, these representations are the only indecomposable representations in the model and the minimal models are examples of *rational* conformal field theories.

At criticality, the restricted solid-on-solid models solved by Andrews, Baxter and Forrester [3,4] offer lattice realisations of the minimal models. Corresponding to each of the irreducible Virasoro representations in a given minimal model, there is a Yang–Baxter integrable boundary condition [5–7] for the lattice realisation: In the continuum scaling limit (or scaling limit, for short), the eigenvalue spectrum of the corresponding transfer matrix (or of the associated Hamiltonian) gives rise to the character of the irreducible representation. In this way, the Hamiltonian of the lattice model becomes the first conformal integral of motion $\mathcal{I}_1 = L_0 - \frac{c}{24}$.

Logarithmic conformal field theory has its roots in work by Rozansky and Saleur [8] and Gurarie [9], but the first thorough analysis of such a theory appeared in a series of papers by Gaberdiel and Kausch [10–12] on a theory with central charge $c = -2$. Their theory is not a minimal model, at least not from the perspective of the Virasoro algebra, but it may be regarded as minimal with respect to an extended symmetry algebra $\mathcal{W}_{1,2}$ related to that of symplectic fermions [13]. The central charge and conformal highest weights of the Virasoro representations are nevertheless of the form (1.1), but with $p = 1$, $p' = 2$ and no upper bounds on the Kac labels r and s. Subsequently, evidence mounted [14–18] suggesting that every Virasoro minimal model can be augmented to a logarithmic conformal field theory of the same central charge. This was made concrete almost ten years ago when such logarithmic models were realised algebraically as conformal field theories with $\mathcal{W}_{p,p'}$ symmetry [19] and conjectured to be the scaling limits of a series of exactly solvable lattice models $\mathcal{LM}(p, p')$ [20]. In these models, the co-prime integers p and p' satisfy $1 \le p < p'$, thus covering the value $c = -2$ (the $\mathcal{W}_{1,p'}$ models were introduced as conformal field theories much earlier [21,22]). We emphasise that the present work deals with the so-called Virasoro picture and thus ignores possible extensions of the Virasoro algebra such as the $\mathcal{W}_{p,p'}$ algebras underlying the W-extended picture [19,23,24].

As lattice theories, the logarithmic minimal models $\mathcal{LM}(p, p')$ describe non-intersecting, densely packed loops on a square lattice. Mathematically, this can be formalised in terms of the Temperley–Lieb algebra $\mathsf{TL}_n(\beta)$ [25], where β denotes the fugacity of the loops and n is the width of the lattice. The models admit infinitely many distinct Yang–Baxter integrable boundary conditions, among which the so-called (r, s)-type, or Kac, boundary conditions play a prominent role. Matrix realisations of the corresponding transfer operators are well-defined, although their construction does not yield representations of the full underlying Temperley–Lieb algebras. It