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Nuclear Physics B 897 (2015) 229–275

NUCLEAR PHYSICS B

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Large-spin and large-winding expansions of giant magnons and single spikes

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Received 21 December 2014; received in revised form 20 April 2015; accepted 21 May 2015

Available online 27 May 2015

Editor: Stephan Stieberger

Abstract

We generalize the method of our recent paper on the large-spin expansions of Gubser–Klebanov–Polyakov (GKP) strings to the large-spin and large-winding expansions of finite-size giant magnons and finite-size single spikes. By expressing the energies of long open strings in $\mathbb{R} \times S^2$ in terms of Lambert's W-function, we compute the leading, subleading and next-to-subleading series of classical exponential corrections to the dispersion relations of Hofman–Maldacena giant magnons and infinite-winding single spikes. We also compute the corresponding expansions in the doubled regions of giant magnons and single spikes that are respectively obtained when their angular and linear velocities become smaller or greater than unity.

1. Introduction and motivation

The exact computation of the full spectrum of the AdS/CFT correspondence [1–3] and the comparison of the scaling dimensions of local operators of planar $\mathcal{N} = 4$ super-Yang–Mills (SYM) theory to the energies of free string states of type IIB superstring theory in $AdS_5 \times S^5$, is the first step towards the determination of the precise relationship between these two theories (that are typically treated as identical in AdS/CFT's strongest formulations). It is therefore very

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http://dx.doi.org/10.1016/j.nuclphysb.2015.05.021

important to identify and study the elementary excitations that string theory in $AdS_5 \times S^5$ and $\mathcal{N}=4$ SYM theory share and are the fundamental building blocks out of which the corresponding spectra may be built.

Giant magnons (GMs) are open, single-spin strings that rotate in $\mathbb{R}\times S^2 \subset AdS_5 \times S^5$. They were found in 2006 by Hofman and Maldacena (HM) [4] and were identified as the string theory duals of magnon excitations of $\mathcal{N}=4$ SYM. Giant magnons are elementary excitations of the IIB Green–Schwarz superstring on $AdS_5 \times S^5$, out of which closed strings and multi-soliton solutions may be formed. The energy–spin relation of a single giant magnon of angular extent $\Delta\varphi$ on a 2-sphere of radius R is:[1]

$$E - J = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{\Delta\varphi}{2}\right|, \quad J=\infty,\ \sqrt{\lambda}=\frac{R^2}{\alpha'}\to\infty, \tag{1.1}$$

where $\Delta\varphi = p$ is the giant magnon's momentum. Superimposing two giant magnons of maximum angular extent $\Delta\varphi=\pi$ gives the Gubser–Klebanov–Polyakov (GKP) closed and folded string that rotates on S^2 [5], the dispersion relation of which is

$$E - J = \frac{2\sqrt{\lambda}}{\pi}, \quad J=\infty,\ \lambda\to\infty. \tag{1.2}$$

According to the AdS/CFT correspondence, the energy E of a string state in $AdS_5\times S^5$ should equal the scaling dimension Δ of its dual $\mathcal{N}=4$ SYM operator. Despite the finiteness of $\mathcal{N}=4$ SYM, its operators typically get renormalized and they thus acquire anomalous dimensions γ, which are the eigenvalues of the gauge theory dilatation operator. The anomalous dimensions may also be found at strong coupling by calculating the energy of their dual strings. Although there exists no systematic way by which to assign a certain gauge theory operator to its dual string state, many such heuristic identifications are known. The above GKP string that rotates inside $\mathbb{R}\times S^2$ for example is dual to the operator $\mathrm{Tr}\left[\mathcal{X}\mathcal{Z}^m\mathcal{X}\mathcal{Z}^{J-m}\right]+\ldots$ of $\mathcal{N}=4$ SYM.

It has been known for quite some time that the one-loop dilatation operator of $\mathcal{N}=4$ SYM theory [6] has the form of an integrable $\mathfrak{psu}(2,2|4)$ spin chain Hamiltonian, which can be diagonalized by means of the Bethe ansatz (BA) [7,8]. An all-loop asymptotic Bethe ansatz (ABA) for the $\mathfrak{su}(2)$ sector of $\mathcal{N}=4$ SYM[(2)] has been proposed by Beisert, Dippel and Staudacher (BDS) [9]. The BDS energy for single magnon states (the elementary spin chain excitations that are dual to GMs) in a spin chain of length $J+1$ is:

$$\epsilon_\infty \equiv E - J = \sqrt{1+\frac{\lambda}{\pi^2}\sin^2\left(\frac{p}{2}\right)}, \quad \lambda = g_{\mathrm{YM}}^2 N, \tag{1.3}$$

where p is the magnon's momentum. At strong 't Hooft coupling ($\lambda\to\infty$) (1.3) gives (1.1) to lowest order and the first quantum correction (aka one-loop shift) vanishes. At weak coupling, the one-loop magnon energy is recovered to lowest order in λ:

$$E - J = 1 + \frac{\lambda}{2\pi^2}\sin^2\left(\frac{p}{2}\right)+\ldots, \quad \lambda\to 0. \tag{1.4}$$

[1] We shall employ the following convention in our paper: $E, J, p=\infty$ and $v,\omega=1$ will denote infinite size (as obtained by computing the $\lim_{J/p\to\infty}$, $\lim_{v/\omega\to 1}$) and $E, J, p\to\infty$, $v,\omega\to 1$ will denote large but still finite size.

[2] The compact $\mathfrak{su}(2)$ sector of $\mathcal{N}=4$ super-Yang–Mills consists of the single-trace operators $\mathrm{Tr}\left[\mathcal{Z}^J\mathcal{X}^M\right]$, where $\mathcal{X},\ \mathcal{Y},\ \mathcal{Z}$ are the three complex scalar fields of $\mathcal{N}=4$ SYM, composed out of the six real scalars Φ of the theory. The $\mathfrak{su}(2)$ sector is dual to (closed) strings that rotate in $\mathbb{R}\times S^3\subset AdS_5\times S^5$ and its one-loop dilatation operator is given by the Hamiltonian of the ferromagnetic $XXX_{1/2}$ Heisenberg spin chain.