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Nuclear Physics B 897 (2015) 374–404

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

String bits and the spin vertex

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Received 19 May 2015; accepted 29 May 2015

Available online 1 June 2015

Editor: Herman Verlinde

Abstract

We initiate a novel formalism for computing correlation functions of trace operators in the planar $\mathcal{N} = 4$ SYM theory. The central object in our formalism is the *spin vertex* which is the weak coupling analogy of the *string vertex* in string field theory. We construct the spin vertex explicitly for all sectors at the leading order using a set of bosonic and fermionic oscillators. We prove that the vertex has trivial monodromy, or put in other words, it is a Yangian invariant. Since the monodromy of the vertex is the product of the monodromies of the three states, the Yangian invariance of the vertex implies an infinite exact symmetry for the three-point function. We conjecture that this infinite symmetry can be lifted to any loop order.

1. Introduction

The old idea of 't Hooft [1] about the possibility of an exact correspondence between the multicolor QCD and some string theory has been realized two decades later for the simpler, conformal invariant "supersymmetric QCD", the maximally super-symmetric Yang–Mills theory [2–4]. Even more excitingly, it has been discovered that the theory is likely to be integrable for all couplings. After a crucial insight by Minahan and Zarembo [5], and a great amount of collective work for one decade (see, for example the review [6]) it became clear that the spectral problem in AdS/CFT can be reformulated in terms of integrable spin chains, for which there exists a package

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http://dx.doi.org/10.1016/j.nuclphysb.2015.05.029
0550-3213/

of well developed mathematical methods originated from the Bethe Ansatz. The spectral problem was formulated very elegantly in terms of the so-called Quantum Spectral Curve [7]. We know in principle how to classify the eigenstates $|\mathcal{H}_\alpha\rangle$ of the dilatation operator, which is represented by the Hamiltonian of the integrable spin chain, and to compute their correlation functions

$$\langle \mathcal{O}_\alpha(x)\mathcal{O}_\beta(y)\rangle = \frac{\delta_{\alpha\beta}\,\mathcal{N}_a}{|x-y|^{\Delta_\alpha+\Delta_\beta}}. \tag{1.1}$$

(It is convenient to use a normalization in which the constants $\mathcal{N}_\alpha$ are given by the norms squared of the corresponding on-shell Bethe states.)

In the last few years the challenge moved from the spectral problem, i.e. the structure of the conformal dilatation operator by which is nowadays considered solved in principle, to the computation of the correlation functions, amplitudes and Wilson loops. Understanding of the structures of these objects in the maximally supersymmetric theory would help devising efficient computation techniques for perturbative QCD. The structure of the interactions is encoded in the operator product expansion

$$\mathcal{O}_\alpha(x)\mathcal{O}_\beta(y) \sim C^{\gamma}_{\alpha\beta}\mathcal{O}_\gamma(y)\,|x-y|^{\Delta_\gamma-\Delta_\alpha-\Delta_\beta}, \tag{1.2}$$

or equivalently, in the three-point function of operators with given conformal weights:

$$\langle \mathcal{O}_\alpha(x)\mathcal{O}_\beta(y)\mathcal{O}_\gamma(z)\rangle = \frac{C_{\alpha\beta\gamma}}{|x-y|^{\Delta_\alpha+\Delta\beta-\Delta_\gamma}|x-z|^{\Delta_\alpha+\Delta_\gamma-\Delta_\beta}|y-z|^{\Delta_\beta+\Delta_\gamma-\Delta_\alpha}}. \tag{1.3}$$

The two sets of constants are related by

$$C^{\gamma}_{\alpha\beta} = C_{\alpha\beta\gamma}/\mathcal{N}_\gamma. \tag{1.4}$$

The structure constants involve trace operators with non-restricted lengths L_1, L_2, L_3. There are two limits in which the problem can be approached by the available techniques, depending on the value of 't Hooft coupling $\lambda \sim g^2_{\text{YM}}N_c$: that of extremely weak coupling $\lambda \to 0$, and that of extremely strong coupling, $\lambda \to \infty$. Furthermore, the methods applied in each of the two limits depend on the values of the spin and the R-charges of the three operators.

At strong coupling, $\lambda \to \infty$, a general framework for computation is given by string theory. The methods depend on the type of operators. The *heavy* operators have large spin of R-charge and correspond to classical strings moving in the AdS space. In the case of three heavy operators, the problem reduces to a generalization of the Plateau problem, namely to find a minimal surface embedded in the AdS background and having prescribed singularities at three punctures. The method to compute the classical action is based on the classical integrability of the string sigma model [8]. Each of the three states represents a classical solution of the sigma model, described by the spectral curve of the classical monodromy matrix. A major ingredient of the method is a condition on the monodromies associated with the three punctures [9]. Namely, the product of the three monodromies must be equal to one, because the path can be contracted, but on other hand it gives a non-trivial information about the solution, which is sufficient to reconstruct the classical action. In [9], the contribution from the AdS_2 part was evaluated for a string rotating only on the sphere. The full problem was solved in [10]. The case of three GKP strings, which requires also a construction of the vertex operators, was solved in [11,12], which led to a remarkably simple formula in terms of contour integrals in the spectral plane.

In the case of two heavy and one light operator, the methods are slightly different, but still based on the integrability. This case was solved in [13,14]. The solution was recently given a major revision in [15], where a missing modular integral was added. This allowed to the authors