Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 897 (2015) 479–499

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Dimensional reduction of the generalized DBI

Jun-Kai Ho [a,c], Chen-Te Ma [a,b,*]

[a] Department of Physics, Center for Theoretical Sciences, Taiwan, ROC
[b] Center for Advanced Study in Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan, ROC
[c] Department of Physics, Brown University, Box 1843, Providence, RI 02912-1843, USA

Received 29 December 2014; received in revised form 23 April 2015; accepted 25 May 2015

Available online 4 June 2015

Editor: Stephan Stieberger

Abstract

We study the generalized Dirac–Born–Infeld (DBI) action, which describes a q-brane ending on a p-brane with a $(q+1)$-form background. This action has the equivalent descriptions in commutative and non-commutative settings, which can be shown from the generalized metric and Nambu-Sigma model. We mainly discuss the dimensional reduction of the generalized DBI at the massless level on the flat spacetime and constant antisymmetric background in the case of flat spacetime, constant antisymmetric background and the gauge potential vanishes for all time-like components. In the case of $q = 2$, we can do the dimensional reduction to get the DBI theory. We also try to extend this theory by including a one-form gauge potential.

1. Introduction

In string theory, the T-duality shows the equivalence of two theories that look different under the exchange of a radii R and α'/R. For the closed strings [1,2], the T-duality of closed strings exchanges winding and momentum modes. In the case of open strings, the T-duality of open strings exchanges the Dirichlet and Neumann boundary conditions. The studies of the T-duality

* Corresponding author.
E-mail addresses: junkai125@gmail.com (J.-K. Ho), yefgst@gmail.com (C.-T. Ma).

http://dx.doi.org/10.1016/j.nuclphysb.2015.05.026

pave the way for the unified theory. One way is to study the low energy effective action. It is well known that the DBI model can be derived from the one-loop β function. The low-energy effective field theory (higher derivative gravity [3] or non-local theory) can be directly found from the one-loop β function. Nevertheless, we encounter the non-geometry or T-fold problem in the massless closed string theory. This problem is generic and unavoidable in string theory. To solve this problem, we need to construct geometric languages to endow string theory with a global geometry. Double field theory [4–19] and generalized geometry [20–22] are two typical examples. This new "stringy geometry" [23,24] geometrizes the non-geometric flux and finds the 10-dimensional supergravity with the non-geometric flux as shown in [25,26]. At the current stage, the most non-trivial problem is that the double field theory needs the section conditions to have gauge invariance. The approaches of relaxing the section conditions can be found in [27–29]. The extension of the α' correction explores a geometric way to find the T-duality with α' correction [30,31]. The geometric structure can also be extended to the 11-dimensional supergravity [32–37]. Some recent good reviews can be found in [38–40].

To get a geometric picture of brane theory, a combination of non-commutative gauge theory and the generalized geometry is necessary. The non-commutative gauge theory of the D-brane is already well known, but the non-commutative gauge theory of the M-brane is still not completely understood [41,42]. Recently, theories based on the equivalence between the commutative and non-commutative gauge theories are found. The theories are the Nambu-Sigma model and generalized DBI model [43,44]. The non-commutative geometry is encoded in the generalized metric, which is an ingredient of the generalized geometry. Although they do not use the full language of the generalized geometry [45,46], they found the evidence for the DBI-like M2–M5 system [47].

The main task of this paper is to calculate the dimensional reduction of the generalized DBI theory at the massless level. We perform the dimensional reduction from a $(q+1)$-brane ending on a $(p+1)$-brane to a q-brane ending on a p-brane. We consider flat spacetime, constant anti-symmetric background field and the $(q+1)$-form gauge field only exists in $(q+1)$-dimensional worldvolume directions (no time direction) in q–p system. The non-trivial result of this theory is that the appearance of the $2(q+1)$-th root, which can be shown by the equivalence between the commutative and non-commutative descriptions, is robust against the dimensional reduction. The most interesting study is the system of a 2-brane ending on a 5-brane. The system can be reduced to a 1-brane ending on a 4-brane by the dimensional reduction. This shows that the system of a 2-brane ending on a 5-brane can be reduced to the DBI theory in our simple consideration. Finally, we discuss the possibility of adding one-form gauge field in the generalized DBI theory. We can include one-form gauge field up to H^2 in principle, and the calculation also demonstrates the potential to extend the generalized DBI theory with different field contents. This study should give the simplest understood of the higher-form fields although it is not a general consideration. Our study on the generalized DBI theory should motivate interest of duality structure in higher dimensions.

The plan of this paper is to first review the generalized DBI theory in Section 2. Then we discuss the dimensional reduction without scalar fields in Section 3 and dimensional reduction with scalar fields in Section 4. Finally, we conclude in Section 5. We also provide the detailed calculation in Appendix A and Appendix B.

2. Review of the generalized DBI

In this section, we follow [43,44] to review the generalized DBI theory. First of all, we show the closed–open string relations from the string sigma model. Secondly, we generalize the