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Two-loop electroweak threshold corrections in the Standard Model

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Abstract

We study the relationships between the basic parameters of the on-shell renormalization scheme and their counterparts in the $\overline{\text{MS}}$ scheme at full order $\mathcal{O}(\alpha^2)$ in the Standard Model. These enter as threshold corrections the renormalization group analyses underlying, e.g., the investigation of the vacuum stability. To ensure the gauge invariance of the parameters, in particular of the $\overline{\text{MS}}$ masses, we work in R_ξ gauge and systematically include tadpole contributions. We also consider the gaugeless-limit approximation and compare it with the full two-loop electroweak calculation.

1. Introduction

The measurement of the Higgs boson mass at the Large Hadron Collider [1] not only fully confirms the validity of the Standard Model (SM) around the electroweak scale, but also opens a possibility for a precise study of the applicability of the SM at energies of the order of the Planck mass. Renormalization group (RG) equations, which determine the dependence on the renormalization scale μ of the running parameters, which are usually defined in the modified minimal-subtraction ($\overline{\text{MS}}$) scheme of dimensional regularization, play an essential rôle in such

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analyses. In the SM, the corresponding RG functions, whose knowledge had been limited to the two-loop order for a very long time [2], have recently been evaluated at three loops for all running parameters, including gauge, Yukawa, and scalar self couplings [3].

The other aspect of the problem is the matching between the $\overline{\rm MS}$ parameters and the physical observables, which gives rise to threshold corrections.[1] These not only include terms of the form $\ln\mu^2$, but also non-logarithmic ones. The relationships between the $\overline{\rm MS}$ and pole masses of the intermediate bosons were obtained at the two-loop level in Refs. [4,5]. As for the threshold corrections to the top and bottom quark masses and Yukawa couplings, the situation is as follows. The QCD corrections, which are dominant, are available at one [6], two [7,8], three [9], and four [10] loops. The two-loop result in the supersymmetric extension of QCD was obtained in Ref. [11]. The one-loop electroweak corrections, of order $O(\alpha)$, were first considered in Ref. [12]. The two-loop mixed $O(\alpha\alpha_s)$ corrections were provided for the bottom quark in Ref. [13] and for the top quark in Refs. [14–16]. Recently, also the two-loop electroweak corrections of order $O(\alpha^2)$ have been obtained in the gaugeless-limit approximation [17]. Also the threshold corrections to the self-coupling constant of the scalar field were intensively studied in the literature. The $O(\alpha)$ corrections were evaluated a long time ago in Ref. [18] and the $O(\alpha\alpha_s)$ ones recently in Ref. [16]. As for the $O(\alpha^2)$ corrections, the leading term was found in Ref. [19], and an interpolation formula, which also includes subleading contributions, was given in Ref. [20]. These analyses were recently revisited in Ref. [21] by providing precise numerical results.

In this paper, we systematically present the complete two-loop threshold corrections, from the orders $O(\alpha)$, $O(\alpha\alpha_s)$, and $O(\alpha^2)$, to all the running parameters of the SM independently obtained by an analytic calculation. This includes the masses of the W, Z, and Higgs bosons (m_W, m_Z, m_H) and those of the top and bottom quarks (m_t, m_b) as well as the gauge couplings (g, g'), the Higgs self-coupling (λ), and the top and bottom Yukawa couplings (y_t, y_b). In contrast to Refs. [20–22], all our calculations are performed in R_ξ gauge keeping the gauge-fixing parameters free, which allows us to explicitly track the ξ dependencies and so to ensure the gauge independence of the threshold corrections and the $\overline{\rm MS}$ parameters. The tadpole diagrams turn out to play a crucial rôle in this (see Subsection 2.2).

This paper is organized as follows. In Section 2, we set up the stage for our calculation of the threshold corrections. In Subsections 2.1–2.4, we discuss the various ingredients entering our analysis. Our results are presented in Section 3 and Appendix A. In Appendix B, we also list the $\overline{\rm MS}$ renormalization constant of the Higgs boson mass.

2. Setup

The SM may exist in two different phases: the symmetric phase and the phase with the spontaneously broken symmetry. The phase is determined by the potential of the scalar field ϕ,

$$V(\phi) = m_\phi^2\,\phi^\dagger\phi + \lambda\,(\phi^\dagger\phi)^2\,, \tag{1}$$

where m_ϕ is a mass parameter and λ is the self-coupling constant of the scalar field. While stability requires $\lambda > 0$, the term m_ϕ^2 can be either positive (symmetric phase) or negative (broken

[1] The usage of the term *threshold corrections* in this context is to indicate that the initial conditions for the RG evolution of the $\overline{\rm MS}$ parameters are determined at some low-lying scale. This term also appears in the effective-field-theory language, where it carries a different meaning.