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Thermodynamics of the topological Kondo model

Francesco Buccheri [a,*], Hrachya Babujian [a,b], Vladimir E. Korepin [a,c], Pasquale Sodano [a,d], Andrea Trombettoni [e,f]

[a] *International Institute of Physics, Universidade Federal do Rio Grande do Norte, 59078-400 Natal, RN, Brazil*

[b] *Yerevan Physics Institute, Alikhanian Brothers 2, Yerevan, 375036, Armenia*

[c] *C. N. Yang Institute for Theoretical Physics, Stony Brook University, NY 11794, USA*

[d] *Departemento de Fisíca Teorica e Experimental, Universidade Federal do Rio Grande do Norte, 59072-970 Natal, RN, Brazil*

[e] *CNR-IOM DEMOCRITOS Simulation Center, Via Bonomea 265, I-34136 Trieste, Italy*

[f] *SISSA and INFN, Sezione di Trieste, Via Bonomea 265, I-34136 Trieste, Italy*

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Abstract

Using the thermodynamic Bethe ansatz, we investigate the topological Kondo model, which describes a set of one-dimensional external wires, pertinently coupled to a central region hosting a set of Majorana bound states. After a short review of the Bethe ansatz solution, we study the system at finite temperature and derive its free energy for arbitrary (even and odd) number of external wires. We then analyse the ground state energy as a function of the number of external wires and of their couplings to the Majorana bound states. Then, we compute, both for small and large temperatures, the entropy of the Majorana degrees of freedom localized within the central region and connected to the external wires. Our exact computation of the impurity entropy provides evidence of the importance of fermion parity symmetry in the realization of the topological Kondo model. Finally, we also obtain the low-temperature behaviour of the specific heat of the Majorana bound states, which provides a signature of the non-Fermi-liquid nature of the strongly coupled fixed point.

* Corresponding author.
E-mail address: buccheri@iip.ufrn.br (F. Buccheri).

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1. Introduction

The possibility of faithfully simulating quantum low dimensional systems gives today the unique opportunity of testing against experiments predictions made by powerful methods – such as bosonization [1] (see also Schulz et al. in [2]), conformal field theory [3], integrable models and Bethe ansatz [4] – developed in theoretical investigations of strongly correlated condensed matter and spin systems. As a result, one may now confidently apply the above methods to newly engineered quantum systems relevant for the fabrication of quantum devices as well as for the realization, in an easily controllable setting, of new phases of matter such as the non-Fermi liquid phases realized in Dirac materials [5] and in overscreened multichannel Kondo models [6]. Within the new models made accessible to theoretical investigations, great opportunities are offered by the analysis of pertinent junctions of one dimensional systems such as ladders and star junctions [7–12].

Networks of one dimensional models attracted much attention over the past few years. In pioneering works [13,14] a star junction of three quantum wires enclosing a magnetic flux was studied: modelling the wires as Tomonaga–Luttinger liquids (TLL), the authors of Ref. [13, 14] were able to show the existence of an attractive finite coupling fixed point, characteristic of the geometry of the circuit. Later, a repulsive finite coupling fixed point was found in a T-junction of one dimensional Bose liquids [15]. Crossed TLL were also the subject of many investigations both analytical [16] and numerical [17]: these analyses pointed out that, in crossed TLLs, the junction induces behaviours similar to those arising from quantum Kondo impurities in condensed matter physics [18]. For what concerns the analysis of junctions of spin chains, in Ref. [19] it was argued that novel critical behaviours emerge when crossing at a point two spin $1/2$ Heisenberg models since, as a result of the crossing, some operators turn from irrelevant to marginal, leading to correlation functions exhibiting power law decays with non-universal exponents. Star junctions of Josephson junction arrays were investigated in [20] with the result that a finite coupling fixed point was also emerging in these superconducting systems. With Majorana fermions [21] star junctions of quantum wires become very attractive since these geometries facilitate their braiding [22] allowing, at least in principle, for the engineering of quantum circuits relevant for the implementation of quantum protocols [23,24].

The very close relation between the phase diagram emerging from the investigation of networks of quantum one-dimensional systems (quantum spin chains and quantum wires, essentially) and the one typical of multichannel Kondo models was established only very recently. In the two papers [25,26] it was shown that a star junction of three critical Ising models and a star junction of three XX models may be made equivalent to the two channel and the four channel over-screened Kondo model, respectively. To achieve their exact mapping, these authors used a generalization of the Jordan–Wigner transformation needed to satisfy the anticommutation relations between fermions located on different legs of the junction. For this purpose, one modifies the usual Jordan–Wigner transformation by the addition of an auxiliary space made – for a star junction of three spin chains – by three Klein factors, i.e. three real anti-commuting fields, lying at the inner boundary of each chain. As a result, in these realizations of the multichannel Kondo model, the central spin, with which the Jordan–Wigner fermions interact, is realized as a non-local combination of three Klein factors. The Kondo effect occurring when bulk fermions scatter on a composite “spin” non-locally encoded by any number of Klein factors located at different space points provides a realization of the topological Kondo effect. Since one expects that an extended spin is less sensitive to noise and decoherence, it is generally believed that this realization