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Entanglement entropy of non-unitary integrable quantum field theory

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Abstract

In this paper we study the simplest massive $1+1$ dimensional integrable quantum field theory which can be described as a perturbation of a non-unitary minimal conformal field theory: the Lee–Yang model. We are particularly interested in the features of the bi-partite entanglement entropy for this model and on building blocks thereof, namely twist field form factors. Non-unitarity selects out a new type of twist field as the operator whose two-point function (appropriately normalized) yields the entanglement entropy. We compute this two-point function both from a form factor expansion and by means of perturbed conformal field theory. We find good agreement with CFT predictions put forward in a recent work involving the present authors. In particular, our results are consistent with a scaling of the entanglement entropy given by $\frac{c_{\text{eff}}}{3}\log\ell$ where c_{eff} is the effective central charge of the theory (a positive number related to the central charge) and ℓ is the size of the region. Furthermore the form factor expansion of twist fields allows us to explore the large region limit of the entanglement entropy and find the next-to-leading order correction to saturation. We find that this correction is very different from its counterpart in unitary models. Whereas in the latter case, it had a form depending only on few parameters of the model (the particle spectrum), it appears to be much more model-dependent for non-unitary models.

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1. Introduction

Entanglement is a fundamental property of quantum systems which relates to the outcomes of local measurements: performing a local measurement may affect the outcome of local measurements far away. This property represents the single main difference between quantum and classical systems. Technological advances have taken entanglement from a strange quantum phenomenon to a valuable resource at the heart of various fields of research such as quantum computation and quantum cryptography. There has also been great interest in developing efficient (theoretical) measures of entanglement, not just in view of the applications above but also as a means to extract valuable information about emergent properties of quantum states of extended systems. One such measure for many-body quantum systems is the bi-partite entanglement entropy (EE) [1], which we will consider here. Other measures of entanglement exist, see e.g. [1–5], which occur in the context of quantum computing, for instance. In its most general understanding, the EE is a measure of the amount of quantum entanglement, in a pure quantum state, between the degrees of freedom associated to two sets of independent observables whose union is complete on the Hilbert space. In the present paper, the two sets of observables correspond to the local observables in two complementary connected regions, A and $\bar{A}$, of a $1+1$-dimensional (1 space + 1 time dimension) extended quantum model, and we will consider cases where the quantum state is the ground state of a *non-unitary*, near-critical model.

Prominent examples of extended one-dimensional quantum systems are quantum spin chains. Their entanglement has been extensively studied in the literature [6–14]. These examples however all refer to unitary quantum spin chains. Interesting examples of non-unitary spin chain systems exist, for instance the famous quantum group invariant integrable XXZ spin chain, with generically non-Hermitian boundary terms; in the thermodynamic limit it has critical points associated with the minimal models of conformal field theory (CFT), including the non-unitary series [15–18]. Another example is provided by the Hamiltonian studied by von Gehlen in [19,20]: the Ising model in the presence of a longitudinal imaginary magnetic field. This Hamiltonian has a critical line (in the phase space of its two couplings) which has been identified with the Lee–Yang non-unitary minimal model of CFT, with central charge $c = -22/5$ [21,22]. In all these examples, the local, extended Hamiltonians are non-Hermitian, yet have *real and bounded energy spectra*. Their critical points are described by CFT models containing non-unitary representations of the Virasoro algebra with real weights, and whose ground states are not the conformal vacua, but negative-weight modules. The reality of the spectra of some non-unitary quantum field theories (including perturbations of the Lee–Yang minimal model) has also been discussed in [23].

Non-Hermitian Hamiltonians with real spectra are the subject of much current research especially in connection with $\mathtt{PT}$-symmetry or pseudo/quasi Hermiticity [24,25] (see [26–28] for reviews and [29] for the interplay with integrability). For instance the critical line of von Gehlen's system [19,20] described above, can be related to $\mathtt{PT}$–symmetry breaking in that it separates the phase space into two regions, one where only real eigenvalues occur, and another where pairs of complex conjugated eigenvalues arise [30]. Experimental studies and theoretical descriptions of new physical phenomena connected to non-Hermitian Hamiltonians have recently emerged, including optical effects [31–33], transitions from ballistic to diffusive transport [34], and dynamical phase transitions [35,36]. Non-Hermitian quantum mechanics is also used in the description of non-equilibrium systems [37], quantum Hall transitions [38], and quantum annealing [39].

At quantum critical points, the scaling limit of the EE has been widely studied within unitary models of CFT [7,8,40–43]. In particular, the combination of a geometric description, Riemann