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The two-photon exchange contribution to muonic hydrogen from chiral perturbation theory

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Abstract

We compute the spin-dependent and spin-independent structure functions of the forward virtual-photon Compton tensor of the proton at $\mathcal{O}(p^3)$ using heavy baryon effective theory including the Delta particle. We compare with previous results when existing. Using these results we obtain the leading hadronic contributions, associated to the pion and Delta particles, to the Wilson coefficients of the lepton–proton four fermion operators in NRQED. The spin-independent coefficient yields a pure prediction for the two-photon exchange contribution to the muonic hydrogen Lamb shift, $\Delta E_{\mathrm{TPE}}(\pi\&\Delta) = 34(13)$ µeV. We also compute the charge, $\langle r^n \rangle$, and Zemach, $\langle r^n \rangle_{(2)}$, moments for $n \geq 3$. Finally, we discuss the spin-dependent case, for which we compute the difference between the four-fermion Wilson coefficients relevant for hydrogen and muonic hydrogen.

1. Introduction

The spin-dependent and spin-independent structure functions of $T^{\mu\nu}$, the forward virtual-photon Compton tensor of the proton, carry important information about the QCD dynamics. They test the Euclidean region of the theory since $Q^2 \equiv -q^2 > 0$. For $Q^2 \sim m_\pi^2 \neq 0$, the behavior of $T^{\mu\nu}$ is determined by the chiral theory, and can be obtained within a chiral expansion using Heavy Baryon Effective Theory (HBET) [1]. If one works within a large N_c ideology (where N_c

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is the number of colours) the Delta particle should be incorporated in the HBET Lagrangian [2], as the Delta and the nucleon become degenerate in the large N_c limit. We use this motivation to incorporate the Delta particle in the effective Lagrangian. We do so along the lines of Refs. [3–5], i.e. we do not impose the large N_c relations among the couplings but let them free and fit to the data. This effective field theory has a double expansion in $\sim m_\pi/m_\rho$ and $\sim\Delta/m_\rho$, where $\Delta = M_\Delta - M_N$. Note that this creates a new expansion parameter $m_\pi/\Delta \sim 1/2$; the associated corrections will be incorporated in our computation together with the pure chiral result.

Within this framework we compute the spin-dependent and spin-independent structure functions of the forward virtual-photon Compton tensor of the proton at $\mathcal{O}(p^3)$ in Heavy Baryon Chiral Perturbation Theory (HBχPT) including the Delta particle. $T^{\mu\nu}$ cannot be directly related to cross sections obtained at fixed energies, as it tests the Euclidean regime. Nevertheless, it is possible to obtain it (up to eventual subtractions) from experiment through dispersion relations, i.e., through specifically weighted averages of measured cross sections over all energies. Possible constructions are the so-called generalized sum rules, which, for large energies, can be related with the deep inelastic sum rules. These have been studied in Ref. [6] for the spin-dependent case. The spin-independent case has been briefly discussed in Ref. [7]. We will not enter into this interesting line of research in this paper.

Instead, our main motivation for obtaining the chiral structure of $T^{\mu\nu}$ is that $T^{\mu\nu}$ appears in the matching computation between HBET and non-relativistic QED (NRQED) that determines $c_3^{pl_i}$ and $c_4^{pl_i}$ ($l_i = e$ or μ), the Wilson coefficients of the lepton–proton four-fermion operators in the NRQED [8] Lagrangian. As soon as hadronic effects start to become important in atomic physics, these Wilson coefficients play a major role. They appear in the hyperfine splitting (spin-dependent) and Lamb shift (spin-independent) in hydrogen and muonic hydrogen (see Refs. [9,10,7]). Therefore, their determination allows us to relate the energy shifts obtained in hydrogen and muonic hydrogen. Even more important, these Wilson coefficients usually carry most of the theoretical uncertainty in these splittings. This is particularly so in the case of the muonic hydrogen Lamb shift. At present, it is the limiting factor for improving the precision of the determination of the electromagnetic proton radius from the measurements taking place at PSI [11,12] of the muonic hydrogen spectra. This necessity to improve our knowledge (of the spin-independent) lepton–proton four-fermion Wilson coefficient has led us to compute this quantity in HBχPT including the Delta particle. Fortunately enough, this object is chiral enhanced. Therefore, the $\mathcal{O}(p^3)$ chiral computation yields a pure prediction, without the need of new counterterms, of $\Delta E_{\rm TPE}$, the (hadronic) two-photon exchange contribution to the muonic hydrogen Lamb shift: $\Delta E_{\rm L} = E(2P_{3/2}) - E(2S_{1/2})$. Note that, since $m_\mu/m_\pi \sim 1$, we keep the complete m_μ/m_π dependence in such predictions. These results have been used in the recent determination of the muonic hydrogen Lamb shift and the proton radius performed in Ref. [13]. One of the main motivations of this paper is to give the details of the hadronic-related part of that analysis.

We profit this analysis to revisit the distinction between the Born and non-Born terms of $T^{\mu\nu}$ and $\Delta E_{\rm TPE}$. Such distinction produces the so-called Zemach (or Born) and polarizability corrections to the Wilson coefficients (names also used for the associated contributions to the energy shifts: hyperfine or Lamb shift). For the spin-independent case we have a good analytical control and can also compute the charge, $\langle r^n\rangle$, and the Zemach, $\langle r^n\rangle_{(2)}$, moments, for $n \geq 3$, since they are dominated by the chiral theory. The polarizability correction of $\Delta E_{\rm TPE}$ is also usually split into the so-called inelastic and subtraction terms. We will also discuss what HBχPT has to say in this respect.

The paper is distributed in the following way. In Section 2 we present HBET and NRQED. In Section 3 we compute $T^{\mu\nu}$. In Section 4 we compute $c_3^{pl_i}$, $\langle r^{2k+1}\rangle$, and $\Delta E_{\rm TPE}$. For the latter we