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Three-loop Higgs self-coupling beta-function in the Standard Model with complex Yukawa matrices

A.V. Bednyakov [a,*], A.F. Pikelner [a], V.N. Velizhanin [b,c]

[a] *Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[b] *Institut für Mathematik und Institut für Physik, Humboldt-Universität zu Berlin, IRIS Adlershof, Zum Großen Windkanal 6, 12489 Berlin, Germany*
[c] *Theoretical Physics Division, Petersburg Nuclear Physics Institute, Orlova Roscha, Gatchina, 188300 St. Petersburg, Russia*

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Abstract

Three-loop renormalization group equations for the Higgs self-coupling and Higgs mass parameter are recalculated in the case of complex Yukawa matrices which encompass the general flavor structure of the Standard Model. In addition, the anomalous dimensions for both the quantum Higgs field and its vacuum expectation value are presented in the $\overline{\text{MS}}$-scheme. A numerical study of the latter quantities is carried out for a certain set of initial parameters.

Keywords: Standard Model; Renormalization group

The discovery of the Higgs boson [1,2] confirms the fact that the Standard Model turns out to be a perfect model describing physics at the electroweak scale. In spite of all attempts to find something beyond the SM, no stringent evidences of new particles were found.

* Corresponding author.
E-mail addresses: bednya@theor.jinr.ru (A.V. Bednyakov), andrey.pikelner@cern.ch (A.F. Pikelner), velizh@thd.pnpi.spb.ru (V.N. Velizhanin).

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Recent analyses [3–6] based on three-loop renormalization group equations [7–9] demonstrated that the SM can be extrapolated up to very high scales without the necessity to introduce additional degrees of freedom.

Unfortunately, current experimental uncertainty in the strong coupling constant and the top quark mass do not allow us to make an accurate prediction whether the SM vacuum is stable only up to $\mathcal{O}(10^{10})$ GeV or up to the Plank scale. It is not surprising that in the above-mentioned studies focused on vacuum stability the flavor structure of the SM was neglected.

In this work, we extend our recent results on Higgs potential parameters to the case of general Yukawa matrices. This kind of result can be important not only in precise studies of vacuum stability, but also in an analysis of different flavor patterns (see, e.g., a review [10]), which can again originate from some New Physics.

The corresponding two-loop expressions [11] can be deduced from the general results of Refs. [12–15]. The three-loop gauge-coupling beta-functions with the full flavor structure were calculated for the first time in Ref. [16] and confirmed later by our group [17]. It should be noted that the expressions presented in this paper *cannot* be obtained from the known results [9,18] in the SM with only one fermion family coupled to the Higgs boson. This is due to the fact that the simple fermion-loop counting and naive generalization of the substitution rules from Refs. [16,17] are not sufficient to distinguish certain Yukawa-matrix traces, which can appear in the final results for the considered quantities (see below). As a consequence, a direct evaluation of Feynman diagrams with explicit flavor indices is required.

For this kind of calculation the Feynman rules for `DIANA` [19], which were used in our previous studies, were appropriately rewritten and a simple routine dealing with explicit flavor indices was developed. In order to validate our codes, we also recalculated the results for the gauge coupling beta-functions, thus confirming the expressions given in Refs. [16,17].

The calculation is carried out in an almost automatic way with the help of the infra-red rearrangement (IRR) [20] procedure implemented in our codes. We start with the Lagrangian of the unbroken SM with the full flavor structure given in our previous paper [17]. For the reader's convenience we present here the terms describing the fermion–Higgs interactions and the Higgs field self-interaction

$$\mathcal{L}_{\text{Yukawa}} = -\big(Y_u^{ij}\big(Q_i^L\Phi^c\big)u_j^R + Y_d^{ij}\big(Q_i^L\Phi\big)d_j^R + Y_l^{ij}\big(L_i^L\Phi\big)l_j^R + \text{h.c.}\big), \tag{1}$$

$$\mathcal{L}_{\text{H}} = (D_\mu\Phi)^\dagger(D_\mu\Phi) - V_H(\Phi), \tag{2}$$

$$V_H(\Phi) = m^2\Phi^\dagger\Phi + \lambda\big(\Phi^\dagger\Phi\big)^2, \qquad \Phi^\dagger\Phi = \frac{h^2+\chi^2}{2} + \phi^+\phi^-. \tag{3}$$

Here λ and $Y_{u,d,l}$ denote the Higgs quartic and Yukawa matrices, respectively. The left-handed quark and lepton SU(2) doublets, Q_i^L, and L_i^L, carry flavor indices $i = 1,2,3$. The same is true for the SU(2) singlets corresponding to the right-handed SM fermions u_i^R, d_i^R, and l_i^R. The Higgs doublet Φ with hypercharge $Y_W = 1$ is decomposed in terms of the component fields:

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{pmatrix}, \qquad \Phi^c = i\sigma^2\Phi^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}(h - i\chi) \\ -\phi^- \end{pmatrix}. \tag{4}$$

The charge-conjugated Higgs doublet Φ^c has $Y_W = -1$ and enters into the Yukawa interactions of the right-handed up-type quarks. We neglect the Higgs mass parameter in the Lagrangian since the corresponding anomalous dimension can be found from the $\overline{\text{MS}}$-renormalization constant of the $|\Phi|^2$ operator (see, e.g., [18,21]).

The utilized IRR prescription consists of the introduction of an auxiliary mass parameter M in every propagator and the subsequent expansion in external momenta