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Exact solution of the one-dimensional Hubbard model with arbitrary boundary magnetic fields

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Abstract

The one-dimensional Hubbard model with arbitrary boundary magnetic fields is solved exactly via the Bethe ansatz methods. With the coordinate Bethe ansatz in the charge sector, the second eigenvalue problem associated with the spin sector is constructed. It is shown that the second eigenvalue problem can be transformed into that of the inhomogeneous XXX spin chain with arbitrary boundary fields which can be solved via the off-diagonal Bethe ansatz method.

Keywords: Hubbard model; Bethe ansatz; $T-Q$ relation

1. Introduction

The Hubbard model is one of the essential models in condensed matter physics. An interesting issue is that the model is exactly solvable in one dimension [1], which provides an important benchmark for understanding the Mott insulators. After Lieb and Wu's pioneering work, a lot of attentions have been paid to the integrability, symmetry [2–5] and physical properties of this

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model. A remarkable result was obtained by Shastry [6] and by Olmedilla et al. [7,8] who constructed the corresponding R-matrix of the one-dimensional Hubbard model and therefore demonstrated its complete integrability in the framework of Yang–Baxter equation [9,10]. Subsequently, the model was resolved [11] via the algebraic Bethe ansatz method based on the result of Shastry. Another interesting issue about this model is the open-boundary problem, which is tightly related to the impurity problem in a Luttinger liquid [12]. The exact solution of the open Hubbard chain was firstly obtained by Shulz [13]. Subsequently, the exact solution of the model with boundary potentials was obtained [14,15]. The integrability of the one-dimensional Hubbard model with diagonal open boundary was demonstrated in [16] by constructing the Lax representation and solved by algebraic Bethe ansatz method [17]. The generic integrable boundary conditions were obtained in [18] by solving the reflection equation [19–21]. It was found [18] that in the spin sector magnetic fields applied on the two end sites do not break the integrability of this model. Although the integrability has been known for long time, the exact solutions (or diagonalization of the Hamiltonian) of the model with arbitrary boundary magnetic fields are still lacking.

In this paper, we study the open Hubbard chain with arbitrary boundary magnetic fields. The Hamiltonian of the model is

$$H = -t\sum_{\alpha,j=1}^{N-1}\left[c_{j,\alpha}^{\dagger}c_{j+1,\alpha}+c_{j+1,\alpha}^{\dagger}c_{j,\alpha}\right]+U\sum_{j=1}^{N}n_{j,\uparrow}n_{j,\downarrow}+h_1^{-}c_{1,\uparrow}^{\dagger}c_{1,\downarrow}+h_1^{+}c_{1,\downarrow}^{\dagger}c_{1,\uparrow}$$
$$+h_1^{z}(n_{1,\uparrow}-n_{1,\downarrow})+h_N^{-}c_{N,\uparrow}^{\dagger}c_{N,\downarrow}+h_N^{+}c_{N,\downarrow}^{\dagger}c_{N,\uparrow}+h_N^{z}(n_{N,\uparrow}-n_{N,\downarrow}), \tag{1.1}$$

where $c_{j,\alpha}^{\dagger}$ and $c_{j,\alpha}$ are the creation and annihilation operators of electrons on site j with spin component $\alpha=\uparrow,\downarrow$; t and U are the hopping constant and the on-site repulsion constant as usual; $n_{j,\alpha}$ are particle number operators, respectively; $\vec{h}_1=(h_1^x,h_1^y,h_1^z)$ and $\vec{h}_N=(h_N^x,h_N^y,h_N^z)$ indicate the boundary fields and $h_j^{\pm}=h_j^x\pm ih_j^y$ for $j=1,N$. We shall show in the following that the model can be exactly solved by combining the coordinate Bethe ansatz and the off-diagonal Bethe ansatz proposed recently in [22–24] for arbitrary $\vec{h}_1$ and $\vec{h}_N$. We remark that the unparallel boundary fields break the $U(1)$ symmetry in spin sector and make the total spin no longer a conserved charge.

The paper is organized as follows. In Section 2, we use the coordinate Bethe ansatz method to derive the eigenvalue equation in the spin sector as that in the periodic case [9]. In Section 3, we transform this eigenvalue problem into that of the inhomogeneous XXX spin chain with boundary fields, which allows us to apply the recently proposed off-diagonal Bethe ansatz method [22–24] to solve it. The exact spectrum of the Hamiltonian and the Bethe ansatz equations are thus obtained. Section 4 is attributed to the reduction to the parallel or anti-parallel boundary case. Concluding remarks are given in Section 5.

2. Coordinate Bethe ansatz

Though the $U(1)$ symmetry in the spin sector is broken by the unparallel boundary fields, the $U(1)$ symmetry in the charge sector is still reserved. The conserved charge corresponding to this reserved symmetry is the total number operator of electrons, namely,

$$\hat{N}=\sum_{j=1}^{N}\{n_{j,\uparrow}+n_{j,\downarrow}\},\quad [H,\hat{N}]=0. \tag{2.1}$$