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The $B \to K^*$ form factors on the lattice

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Abstract

The extraction of the $B \to K^*$ transition form factors from lattice data is studied, applying non-relativistic effective field theory in a finite volume. The possible mixing of πK and ηK states is taken into account. The two-channel analogue of the Lellouch–Lüscher formula is reproduced. Due to the resonance nature of the K^*, an equation is derived, which allows to determine the form factors at the pole position in a process-independent manner. The infinitely-narrow width approximation of the results is discussed.

1. Introduction

Rare B decay modes provide one of the best opportunities in the search for physics beyond the Standard Model (BSM). Among them, $B \to K^* l^+ l^-$ is regarded as one of the most important channels, as the polarization of the K^* allows a precise angular reconstruction resulting in many observables which can be tested in the Standard Model (SM) and its extensions [1–6]. In 2013,

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LHCb [7] published the first analysis of a set of optimized observables, presenting an interesting pattern of deviations, confirmed by later measurements with a larger statistics [8], as well as by a recent analysis from the Belle Collaboration [9]. A first interpretation of this pattern of deviation was proposed [10], where the Wilson coefficient C_9 of the pertinent semileptonic operator (and, possibly, other coefficients as well), received contribution from the BSM physics. Further experimental results have indicated deviations concerning the branching ratios of $B \to K^*\mu^+\mu^-$, but also $B_s \to \phi\mu^+\mu^-$ and $B \to K\mu^+\mu^-$, with the possibility of a violation of lepton flavor universality between electron and muon modes [11–13]. These results triggered lots of activities on the theoretical side and, in particular, their consequences on global fits are being studied [14–16]. In these global fits, a special attention has to be paid to the theoretical uncertainties arising from the *form factors* of the corresponding hadronic matrix elements, which affect the branching ratios involved in the fit. In the low recoil region, which will be our main focus here, these form factors are mostly known from light cone sum rules, which suffer from relatively large uncertainties [17,18]. It would thus be particularly interesting to have information on these quantities from lattice QCD simulations. Also, the method used to calculate these form factors could be applied to other interesting processes as, for example, $B \to K^*\gamma$.

Recently, the first unquenched lattice QCD calculations of the $B \to K^*$ form factors have appeared [19–21] (see also Refs. [22–28] for quenched results). Although this work represents a major progress in the field, the simulations have been performed at such quark mass values that the $K^*(892)$ resonance has been treated as a stable particle. Correspondingly, the standard methods of the lattice QCD could be used for the analysis of the data. However, they are not applicable anymore, when the K^* eventually decays into πK.

The following question has to be addressed: how to compute the matrix elements involving two strongly interacting particles in the in- or out-state? Briefly, the answer is given by the so-called Lellouch–Lüscher method [29]. It is a generalization of the Lüscher finite-volume approach [30], which provides a method to extract the elastic phase shifts and the resonance parameters (the mass and width) from the two-particle discrete energy levels spectrum, measured on the lattice.

At the next step, it should be understood, how to *define* the matrix elements involving resonances such as K^*, ρ, or Δ. As it has been argued in Refs. [31,32], the only plausible field-theoretical definition necessitates an analytic continuation of the matrix element to the resonance pole position in the complex plane. Therefore, strictly speaking, the corresponding form factor can only be defined at the resonance pole. The other well known definition of the form factor is based on the Breit–Wigner parameterization of the resonant amplitude (see, e.g., Refs. [33,34]). However, this definition yields a model- and process-dependent result, since the background is unknown. If the width of the resonance is not very small (it is roughly 50 MeV in the case of the $K^*(892)$), using different definitions might have an effect on the extracted observables.

There is an additional effect, which is due to the presence of the ηK threshold. For physical quark masses, it is approximately 150 MeV above the K^* mass, and this value will be reduced when the light quark masses, used in the simulations, are higher. One could expect that the effect of this threshold might be seen in the data. The recent lattice calculation by the Hadron Spectrum Collaboration, however, indicates that the coupling between the ηK and πK channels remains small even at the pion mass as large as roughly 400 MeV [35,36]. Nevertheless, the two-channel problem has to be addressed. Although of academic interest in the present context, a similar theoretical framework could be useful, e.g., for the lattice extraction of the electromagnetic form factors of the $\Lambda(1405)$ resonance (see Refs. [37,38] for the recent lattice results).