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$I = 1$ and $I = 2$ π–π scattering phase shifts from $N_f = 2 + 1$ lattice QCD

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Abstract

The $I = 1$ p-wave and $I = 2$ s-wave elastic π–π scattering amplitudes are calculated from a first-principles lattice QCD simulation using a single ensemble of gauge field configurations with $N_f = 2 + 1$ dynamical flavors of anisotropic clover-improved Wilson fermions. This ensemble has a large spatial volume $V = (3.7\ \text{fm})^3$, pion mass $m_\pi = 230$ MeV, and spatial lattice spacing $a_s = 0.11$ fm. Calculation of the necessary temporal correlation matrices is efficiently performed using the stochastic LapH method, while the large volume enables an improved energy resolution compared to previous work. For this single ensemble we obtain $m_\rho / m_\pi = 3.350(24)$, $g_{\rho\pi\pi} = 5.99(26)$, and a clear signal for the $I = 2$ s-wave. The success of the stochastic LapH method in this proof-of-principle large-volume calculation paves the way for quantitative study of the lattice spacing effects and quark mass dependence of scattering amplitudes using state-of-the-art ensembles.

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1. Introduction

Hadron–hadron scattering amplitudes are of central importance in the phenomenology of QCD and confining scenarios of Beyond-the-Standard Model (BSM) physics. While Euclidean lattice gauge simulations are a proven first-principles approach for these theories, the calculation of hadron–hadron scattering on the lattice has long been a challenge. First and foremost, the Maiani-Testa No-Go Theorem demonstrates that on-shell amplitudes cannot (in general) be directly obtained from Euclidean space matrix elements [1]. This difficulty was overcome by Lüscher's relation between elastic scattering phase shifts and the deviation of finite-volume two-hadron energy spectra from their non-interacting values [2].

While this relation has been known since the early 90s, only recently are lattice QCD calculations of scattering amplitudes starting to have sufficient statistical precision and energy resolution to clearly identify resonance features. This delay is mostly due to the difficulty in precisely calculating temporal correlation functions

$$C_{ij}(t-t_0) = \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(t_0)\rangle = \sum_n A_{ni} A^*_{nj} \mathrm{e}^{-E_n(t-t_0)}, \tag{1.1}$$

where $\hat{\mathcal{O}}_i$ and $\hat{\mathcal{O}}_j$ are suitable interpolating operators with the quantum numbers of interest and the sum is over all finite-volume energy eigenstates. After calculating such correlation functions on a gauge field ensemble, the finite-volume energies $\{E_n\}$ are extracted from their temporal fall-off.

To obtain finite-volume two-hadron energies, correlation functions between two-hadron interpolating operators are required. These two-hadron correlation functions in turn typically require the evaluation of valence-quark-line-disconnected Wick contractions[1] and contain interpolating operators which annihilate states with definite momentum. After integration over the Grassmann-valued quark fields, this requires the quark propagator from all space–time points to all space–time points. As the quark propagator is the inverse of the large-dimension and ill-conditioned Dirac matrix, these 'all-to-all' propagators (and thus multi-hadron correlation functions) are naively intractable. Inversion of the Dirac matrix M is performed by solving the linear system $M\phi = \eta$ for multiple right-hand sides and is typically the dominant cost in calculating fermionic correlation functions. The solution of this system for each spacetime point is not feasible, preventing the naive approach to all-to-all quark propagators.

However, substantial progress has been made by treating quark propagation only in the subspace spanned by the lowest-lying eigenmodes of the three-dimensional gauge-covariant Laplace operator [3]. Apart from facilitating the evaluation of these correlation functions, this 'distillation' procedure has the added benefit of reducing the contamination of unwanted excited states. It can thus be viewed as a form of 'quark smearing', a common procedure used in lattice QCD to reduce the contribution of higher terms in Eq. (1.1) by suppressing their overlaps. The spatial profile of this smearing wavefunction is approximately gaussian with a width controlled by the number of low-lying Laplacian eigenmodes retained in the projection (N_v).

The cutoff eigenvalue therefore defines the smearing wavefunction and must be fixed in physical units. Unfortunately, if the cutoff eigenvalue is held fixed the number of eigenmodes in this subspace increases proportionally to the spatial volume. The distillation approach requires a number of Dirac matrix inversions $N_D \propto N_v$ which results in an unfavorable volume scal-

[1] 'Disconnected' Wick contractions are those in which quark fields at the same time are contracted.