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$\mathcal{N} = 2$ supersymmetric odd-order Pais–Uhlenbeck oscillator

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Abstract

We consider an $\mathcal{N} = 2$ supersymmetric odd-order Pais–Uhlenbeck oscillator with distinct frequencies of oscillation. The technique previously developed in [Bolonek and Kosiński (2005) [7]], [Masterov (2016) [10]] is used to construct a family of Hamiltonian structures for this system.

1. Introduction

A systematic way to construct a Hamiltonian formulation for nondegenerate higher-derivative mechanical systems is based on Ostrogradsky's approach [1]. Canonical formalism for degenerate higher-derivative models can be obtained with the aid of Dirac's method for constrained systems [2] or by applying the Faddeev–Jackiw prescription [3].

However, some higher-derivative models are multi-Hamiltonian. The simplest example of such systems is the one-dimensional fourth-order Pais–Uhlenbeck (PU) oscillator [4]. Ostrogradsky's Hamiltonian of this system is unbounded from below. As a consequence, quantum theory of the model faces ghost-problem (see, e.g., a detailed discussion in Ref. [5]). For distinct frequencies of oscillation, this Hamiltonian can be presented as a difference of two harmonic

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oscillators by applying an appropriate canonical transformation [4,6]. This representation provides two functionally independent positive-definite integrals of motion. As was observed in [7] (see also Ref. [8]), a linear combination involving arbitrary nonzero coefficients of these constants of motion can also play a role of a Hamiltonian for the fourth-order PU oscillator.[1] Thus, for positive coefficients, the alternative Hamiltonian is positive-definite and consequently is more relevant for quantization than Ostrogradsky's one.

For arbitrary odd and even orders, the PU oscillator with distinct frequencies of oscillation can also be treated by the technique employed in Ref. [7]. This fact has been established in [10, 11] (see also Ref. [12]) where the corresponding families of Hamiltonian structures have been constructed. The main advantage of the alternative Hamiltonian formulation obtained in such a way is that this may correspond to a positive-definite Hamiltonian.

The even-order PU oscillator with distinct frequencies of oscillation admits an $\mathcal{N}=2$ supersymmetric extension [13]. This generalization is invariant under the time translations. However, the Noether charge associated with this symmetry can be presented as a sum of $\mathcal{N}=2$ supersymmetric harmonic oscillators which alternate in a sign [13] (see also Ref. [14]). A canonical formalism with regard to a such Hamiltonian brings about trouble with ghosts upon quantization [13]. This problem is reflected in the fact that the quantum state space of the model contains negative norm states, while a ground state is absent. In Ref. [10] an alternative Hamiltonian formulation for an $\mathcal{N}=2$ supersymmetric even-order PU oscillator has been constructed so as to avoid these nasty features.

For a particular choice of oscillation frequencies, an $\mathcal{N}=2$ supersymmetric extension of the odd-order PU oscillator has been derived in Ref. [15]. It has been shown that this extension accommodates conformal symmetry provided frequencies of oscillation form a certain arithmetic sequence. Any other aspects related with the $\mathcal{N}=2$ supersymmetric odd-order PU oscillator remain completely unexplored. In particular, a canonical formulation of this model has not yet been considered. The purpose of the present work is to construct a Hamiltonian formulation for an $\mathcal{N}=2$ supersymmetric odd-order PU oscillator with distinct frequencies of oscillation by applying the technique previously developed in Refs. [7,10].

The paper is organized as follows. In the next section we consider the odd-order PU oscillator with distinct frequencies of oscillation and introduce an $\mathcal{N}=2$ supersymmetric extension of this model. A Hamiltonian formulation for an $\mathcal{N}=2$ supersymmetric third-order PU oscillator is constructed in Sect. 3, while the general case is treated in Sect. 4. In Sect. 5, a quantum version of the $\mathcal{N}=2$ supersymmetric odd-order PU oscillator is considered. We summarize our results and discuss further possible developments in the concluding Sect. 6. Some technical details are given in Appendix. Throughout the work summation over repeated spatial indices is understood, unless otherwise is explicitly stated. Both a superscript in braces and a number of dots over spatial coordinates designate the number of derivatives with respect to time. Complex conjugation of a function f is denoted by f^*. Hermitian conjugation of an operator $\hat{a}$ is designated as $(\hat{a})^\dagger$.

2. The model

Symmetries of the PU oscillator have recently attracted some attention [16–23]. The interest was motivated by the desire to realize the so-called l-conformal Newton–Hooke algebra [24–26]

[1] An alternative Hamiltonian formulation for the fourth-order PU oscillator has been also constructed in paper [9].