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Wronskians, dualities and FZZT-Cardy branes

Chuan-Tsung Chan [a,*], Hirotaka Irie [a,b], Benjamin Niedner [c], Chi-Hsien Yeh [d,e]

[a] *Department of Applied Physics, Tunghai University, Taichung 40704, Taiwan*
[b] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[c] *Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, UK*
[d] *National Center for Theoretical Sciences, National Tsing-Hua University, Hsinchu 30013, Taiwan*
[e] *Department of Physics and Center for Advanced Study in Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan, ROC*

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Abstract

The resolvent operator plays a central role in matrix models. For instance, with utilizing the loop equation, all of the perturbative amplitudes including correlators, the free-energy and those of instanton corrections can be obtained from the spectral curve of the resolvent operator. However, at the level of non-perturbative completion, the resolvent operator is generally *not* sufficient to recover all the information from the loop equations. Therefore it is necessary to find *a sufficient set of operators* which provide the missing non-perturbative information.

In this paper, we study *generalized Wronskians* of the Baker–Akhiezer systems as a manifestation of these new degrees of freedom. In particular, we derive their isomonodromy systems and then extend several spectral dualities to these systems. In addition, we discuss how these Wronskian operators are naturally aligned on the Kac table. Since they are consistent with the Seiberg–Shih relation, we propose that these new degrees of freedom can be identified as FZZT-Cardy branes in Liouville theory. This means that FZZT-Cardy branes are the bound states of *elemental FZZT branes* (i.e. the twisted fermions) rather than the bound states of principal FZZT-brane (i.e. the resolvent operator).

* Corresponding author.
E-mail addresses: ctchan@thu.edu.tw (C.-T. Chan), irie@yukawa.kyoto-u.ac.jp, hirotaka.irie@gmail.com (H. Irie), benjamin.niedner@physics.ox.ac.uk, benjamin.niedner@gmail.com (B. Niedner), d95222008@ntu.edu.tw (C.-H. Yeh).

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1. Introduction and summary

Matrix models are tractable toy models for studying non-perturbative aspects of string theory [1–53]. Not only their perturbation theory is controllable to all orders [45,46] (including correlators, free-energy and those of instanton corrections) but also non-perturbative completions (such as their associated Stokes phenomena) have revealed intriguing and solvable features in these models [50–53]. Such solvable aspects of matrix models have pushed forward our fundamental understanding of string theory beyond the perturbative perspectives. Although its beautiful framework is already considered to be "well-established", it is our belief that solvability of matrix models is now opening the next stage of development toward a non-perturbative definition of string theory.

In this paper, we will explore a new horizon toward the fundamental understanding of such solvable aspects. The key question which we would like to address is *what are the non-perturbative degrees of freedom in string theory/matrix models?* We now have quantitative control over non-perturbative completions of matrix models (especially by the theory of isomonodromy deformations [57]). So we would like to investigate how the non-perturbative completions help us in identifying the fundamental degrees of freedom of matrix models and in extracting the physical consequence of their existence.

Among various aspects of the solvability, we would like to consider the formulation of *the loop equations* [2,4]. In this formulation, *the resolvent operator* has been the central player in the past studies [2]. In particular, what we try to tackle is the folklore (or a commonsense) about the resolvent operator: the resolvent operator is believed to contain all the information of matrix models and is considered as the only fundamental degree of freedom. However, as it has been noticed in several non-perturbative analyses [50–53], the resolvent operator is not sufficient to *fully* describe the system non-perturbatively: *There are missing degrees of freedom* which are necessary in order to complete all the information of the matrix models. This new type of degrees of freedom is the main theme of this paper. In fact, as far as the authors know, the study on this aspect is almost missing in the literature but it should push forward our understanding of matrix models toward new regimes of non-perturbative study.

The rest of this introduction is organized as follows: We first discuss *why the resolvent operator is not sufficient* in Section 1.1. We then discuss *what are the missing degrees of freedom and how they are described* in Section 1.2. In fact, the developments in non-critical string theory [58], especially of Liouville theory [58–66], come out to provide an interesting clue. Our short answer to the question is that they are *Wronskians of the Baker–Akhiezer systems*. This Wronskian construction is motivated from *the FZZT-Cardy branes* [65] in Liouville theory. Accordingly, we will encounter three variants of Wronskians. In Section 1.2, therefore, we briefly present *the whole picture of our proposal and the roles of these variants of Wronskians* as a summary of this paper. The organization of this paper is presented in Section 1.3.

Although this paper focuses only on the (double-scaled) two-matrix models which describe (p,q) minimal string theory, the fundamental ideas can also be applied to any other kinds of matrix models. There is no big difference at least if they are described by the various types of topological recursions [45,46,67–71].