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Exploring Lovelock theory moduli space for Schrödinger solutions

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Abstract

We look for Schrödinger solutions in Lovelock gravity in $D > 4$. We span the entire parameter space and determine parametric relations under which the Schrödinger solution exists. We find that in arbitrary dimensions pure Lovelock theories have Schrödinger solutions of arbitrary radius, on a co-dimension one locus in the Lovelock parameter space. This co-dimension one locus contains the subspace over which the Lovelock gravity can be written in the Chern–Simons form. Schrödinger solutions do not exist outside this locus and on this locus they exist for arbitrary dynamical exponent z. This freedom in z is due to the degeneracy in the configuration space. We show that this degeneracy survives certain deformation away from the Lovelock moduli space.

1. Introduction

The Schrödinger solutions belong to a class of solutions to the gravitational equations of motion which asymptotically do not preserve the Lorentz symmetry. They, however, do respect some non-relativistic symmetries. The deviation from the relativistic symmetry is parametrized by the Schrödinger scaling exponent z, or the dynamical exponent.

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The Schrödinger solution was first obtained by Son [1] as well as Balasubramanian and Mcgreevy [2]. They assumed the stress tensor consisting of the cosmological constant term and the pressure-less dust. The Schrödinger solution possesses the Galilean boost invariance by assigning a specific transformation property to one of the light-like directions [1,2] (non-relativistic metrics in higher derivative gravity were discussed in [3]).

In this note we analyze this question in more detail by spanning the entire coupling parameter space of Lovelock theories in various dimensions. Up to four space–time dimensions, the Lovelock action is identical to the Einstein Hilbert action with the cosmological constant, but from five dimensions onwards the Lovelock action has additional Gauss–Bonnet term in the action. This term can be added in four dimensions as well but being total derivative term it does not contribute to the dynamics. In five and higher dimensions the Gauss–Bonnet term does contribute to the dynamics. Similarly the cubic order Lovelock term can be added from six dimensions onwards but it contributes to dynamics only from seven dimensions onwards.

We will show that the Schrödinger metric is generically not a solution to the Lovelock equations of motion, however, it exists as a solution on a co-dimension 1 locus in the Lovelock coupling space. We show that the Schrödinger solution exists precisely on the same locus on which the Lifshitz solution is known to exist.[1] In our computation we restrict ourselves to the Lovelock terms up to cubic order in the curvature tensor but we generalize our analysis to arbitrary dimensions. The co-dimension 1 locus on which we get the Schrödinger solution is interesting from another point of view. It is known that the Lovelock theories can be written in terms of the parity preserving Chern–Simons theory. However, this representation exists only for specific values of the Lovelock couplings. The Chern–Simons formulation exists at a point on this co-dimension 1 locus on which we find the Schrödinger solutions. We present these solutions in the Chern–Simons gauge field forms as well.

The Schrödinger solutions are relevant from the point of view of application to holographically dual condensed matter physics systems. It then naturally raises a question of relevance of these higher dimensional solutions to $2+1$ and $3+1$ dimensional condensed matter systems. In this regard it is worth pointing out that unlike the AdS and Lifshitz holography which relates D dimensional theory of gravity to $D-1$ dimensional field theory, the Schrödinger holography relates D dimensional theory of gravity to $D-2$ dimensional field theory. Therefore, $4+1$ and $5+1$ dimensional Lovelock theories are relevant to $2+1$ and $3+1$ dimensional boundary physics. Higher dimensional theories can be dimensionally reduced to lower dimensional theories. Such higher dimensional theories typically give rise to scalar-tensor theories of gravity which are either referred to as Galileon or Horndeski theories [5–8]. For example, let us consider $D=d+n+1$ dimensional theory of gravity with the cosmological constant, the Einstein–Hilbert, the Gauss–Bonnet term

$$S=\int d^Dx\sqrt{-g}\,[R-2\Lambda+a_2\mathcal{L}_2]\ , \tag{1.1}$$

where $\mathcal{L}_2$ is the Gauss–Bonnet term. We will dimensionally reduce it down to $d+1$ dimensions by using an n-dimensional compact manifold $\tilde{K}_n$ such that

$$ds_D^2=d\bar{s}_{d+1}^2+e^{\phi}d\tilde{K}_n^2\ . \tag{1.2}$$

This is a simple but consistent diagonal toroidal compactification which gives rise to one extra scalar degree of freedom, that is the size of the internal space. All terms with a tilde refer to

[1] For closely related solutions of Kasner type in the Lovelock theory, see [4].