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String states, loops and effective actions in noncommutative field theory and matrix models

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Abstract

Refining previous work by Iso, Kawai and Kitazawa, we discuss bi-local string states as a tool for loop computations in noncommutative field theory and matrix models. Defined in terms of coherent states, they exhibit the stringy features of noncommutative field theory. This leads to a closed form for the 1-loop effective action in position space, capturing the long-range non-local UV/IR mixing for scalar fields. The formalism applies to generic fuzzy spaces. The non-locality is tamed in the maximally supersymmetric IKKT or IIB model, where it gives rise to supergravity. The linearized supergravity interactions are obtained directly in position space at one loop using string states on generic noncommutative branes.

1. Introduction

Noncommutative field theory (NCFT) was conceived as a generalization of (quantum) field theory to noncommutative or quantized spaces. One of the early hopes was that the intrinsic uncertainty scale of the geometry would lead to a UV regularization of the corresponding field theory. However, it turned out that this is not the case. Rather, the phenomenon of UV/IR mixing [1] leads to an unexpected behavior of the quantum effective action at low energies, and IR divergences arise due to UV contributions in the loops. This phenomenon was partially understood from various points of view, see e.g. [2–9] and references therein. The realization of noncommu-

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tative field theory in string theory [10] suggested an interpretation in terms of a closed string exchange [5], a geometric understanding in terms of emergent gravity was found [11], and a relation with non-locality was exhibited [3,4,12,13]. In any case, UV/IR mixing means that noncommutative field theory is not simply a deformation of ordinary field theory, but is qualitatively different.

In this paper, we consider a powerful tool in the framework of noncommutative field theory given by string states, refining and developing the ideas introduced in [12]. These states make the string-like character of NC field theory manifest, they provide a clear understanding of UV/IR mixing, and an efficient way to compute loop integrals. String states are defined as $|x\rangle\langle y| \in End(\mathcal{H})$, in terms of coherent states $|x\rangle$ on the noncommutative space under consideration. They are elements of the noncommutative algebra of functions on the space, but they have no classical analog in field theory. They play a dominant role in the loop integrals, which explains the stringy nature of NCFT.

One of the technical results of this paper is a representation of one-loop integrals on fuzzy spaces in terms of integrals over string states rather than group-theoretical harmonics. This was developed to find a practical way of evaluating loop corrections on such backgrounds in Yang–Mills matrix models. The standard way of evaluating these loop integrals is to use a group-theoretical basis of functions (such as spherical harmonics on the fuzzy sphere). However, this leads to unreasonable difficulties, requiring the asymptotics of various group-theoretical objects such as 6J symbols and their higher analogs. Moreover on generic spaces without symmetry, such a computation was practically impossible outside of the semi-classical regime. Most importantly, the group-theoretical approach hides the physical meaning of the results. Although the main ideas of the present approach are contained in [12], we improve their results by replacing the ad-hoc lattices by an integration in position space, which yields a simple closed formulas for the effective action in position space.

We first review the basic facts about coherent states on the fuzzy sphere, which generalize to any quantized compact coadjoint orbit. In particular, the separation of the space of function into the semi-classical IR regime and the – much larger – UV regime is carefully discussed. The latter is best described by the string states, which are interpreted as strings whose energy and momentum is given by their length. These states have the remarkable property that they (approximately) diagonalize the Laplacian, and are “bi-local” in configuration space. The corresponding propagator takes a very simple form, which makes them ideally suited for quantization. An over-completeness relation leads to an exact representation of the trace in the one-loop effective action. We apply this in the basic one-loop integrals, and obtain a closed form for the (quadratic) 1-loop effective action in position space. This works for any quantized coadjoint orbit, and reproduces the known results for the fuzzy sphere which were obtained originally in a more complicated and less transparent way. On the Moyal–Weyl quantum plane, the origin of the non-local UV/IR mixing is clarified. The generalization to generic fuzzy spaces and to higher-loop computations is also discussed.

The results clearly exhibit the non-local nature of generic noncommutative field theories at the quantum level, making the previous observations in [3,4,13] more explicit and manifest. Hence attempts to directly use generic (non-supersymmetric) NC field theories as a replacement for ordinary local QFT are doomed,[1] and only the maximally supersymmetric model(s) remain as candidates for a fundamental, “UV-complete” quantum theory.

[1] One may however consider various limits of noncommutative field theories, which may again become local, see e.g. [14].