Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 910 (2016) 431–457
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Matrix factorizations and elliptic fibrations

Harun Omer

Received 11 May 2016; received in revised form 23 June 2016; accepted 11 July 2016
Available online 18 July 2016
Editor: Stephan Stieberger

Abstract

I use matrix factorizations to describe branes at simple singularities of elliptic fibrations. Each node of the corresponding Dynkin diagrams of the ADE-type singularities is associated with one indecomposable matrix factorization which can be deformed into one or more factorizations of lower rank. Branes with internal fluxes arise naturally as bound states of the indecomposable factorizations. Describing branes in such a way avoids the need to resolve singularities. This paper looks at gauge group breaking from E_8 fibers down to $SU(5)$ fibers due to the relevance of such fibrations for local F-theory GUT models. A purpose of this paper is to understand how the deformations of the singularity are understood in terms of its matrix factorizations. By systematically factorizing the elliptic fiber equation, this paper discusses geometries which are relevant for building semi-realistic local models. In the process it becomes evident that breaking patterns which are identical at the level of the Kodaira type of the fibers can be inequivalent at the level of matrix factorizations. Therefore the matrix factorization picture supplements information which the conventional less detailed descriptions lack.

1. Introduction

Historically, the idea of a category-theoretical description of string theory even predates the discovery of D-branes and first appeared in the context of homological mirror symmetry, where Kontsevich conjectured that the Fukaya category of a Calabi–Yau is equivalent to the derived category of coherent sheaves of the mirror Calabi–Yau [1]. Later it was suggested that D-branes can be described as sheaves [2] and brane/anti-brane systems were described as derived categories [3]. It was in the context of topological boundary Landau–Ginzburg models that D-branes

E-mail address: harunomer@gmail.com.

http://dx.doi.org/10.1016/j.nuclphysb.2016.07.013

have been explicitly described as and written out as matrix factorizations, using a result of Eisenbud [4]. A selection of papers with particular emphasis on review articles to introduce the subject is found in the references [5–14]. The category-theoretical description is powerful and its application extends beyond Landau–Ginzburg (LG) models. A strength of the description is its ability to deal with branes at singularities naturally without the need to resolve the geometry. This comes particularly handy in F-theory where 7-branes are located at geometric singularities arising from elliptic fibrations [15–28]. In particular it has been shown that models with phenomenologically viable features can be built only around a single singularity. This singularity can be thought of as the location of a 7-brane with GUT gauge group, whose singularity type is further enhanced as it intersects other branes at the location of the singularity. In such local F-theory models it is sufficient to focus on the vicinity of the singularity in order to derive physical properties of the theory. Numerous papers have been published in this direction in the last few years.

It is to date an open problem to fully characterize the (p, q)-branes arising in F-theory, the strong coupling limit of type IIB theory. F-theory treatments of 7-branes are usually limited to describing the Kodaira fiber type and the behavior under monodromies. This paper does not fully remedy the situation, however the formulation in terms of matrix factorizations does encode additional information as it is quite generic. The category of matrix factorizations is isomorphic to the derived category of coherent sheaves. Mathematically, a sheaf tracks local data on the open sets of a topological space. In physics a sheaf is often thought of as a brane with a gauge bundle. Sheaves together with morphisms form a category. Morphisms are defined to contain an identity element and satisfy the associativity property $h \circ (g \circ f) = (h \circ g) \circ f$, which reflects the recombination property of open strings. Consequently matrix factorizations capture rather generic topological properties of branes and strings, avoiding specialized or restrictive assumptions. To the extent that in F-theory any of these properties are violated or are insufficient, the present approach also finds its limits. Nevertheless, the description encodes more information than just the fiber type. Collinucci and Savelli state “We will argue that the complete way to specify an F-theory compactification is to define a geometry, and a corresponding choice of matrix factorization” [29]. In their work [30,29], they sought to rederive a theoretical foundation for applying matrix factorizations to F-theory. The authors use simple toy models of matrix factorizations. One goal of this paper is to make available complete sets of indecomposable factorizations for any given fibration. Neither in the physics nor in the mathematics literature exist systematic parameter-dependent factorizations of the fiber equations. Individual factorizations for singularities such as E_8 and E_7 can look rather similar, but that holds only after a suitable similarity transformation and no efforts have so far been made to write all factorizations in a consistent manner. Furthermore, as will be seen, there can be more than one way to deform a set of factorizations for a given ADE group into a set of factorization of a lower rank group. This should be thought of as reflecting the way how the deeper geometric structure of the singularity is deformed.

For a formal discussion to the connection to F-theory to the extent that it is known, see the work of Collinucci and Savelli. Readers more familiar with or more interested on the Landau–Ginzburg (LG) models can continue to think in terms of them. LG models are valued in a weighted projective target space defined by some equation $W = 0$ where W is the superpotential of the LG model. LG models with singularities, namely the ADE minimal models, have been the subject of extensive research and LG realizations of matrix factorizations with a torus as target space have also been discussed at length, for instance in [31–34]. The torus can be parameter-dependent and degenerate at certain regions of the parameter space, without affecting the description as matrix factorization. In principle, the branes defined by the factorizations of W