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Universal corner contributions to entanglement negativity

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Abstract

It has been realised that corners in entangling surfaces can induce new universal contributions to the entanglement entropy and Rényi entropy. In this paper we study universal corner contributions to entanglement negativity in three- and four-dimensional CFTs using both field theory and holographic techniques. We focus on the quantity χ defined by the ratio of the universal part of the entanglement negativity over that of the entanglement entropy, which may characterise the amount of distillable entanglement. We find that for most of the examples χ takes bigger values for singular entangling regions, which may suggest increase in distillable entanglement. However, there also exist counterexamples where distillable entanglement decreases for singular surfaces. We also explore the behaviour of χ as the coupling varies and observe that for singular entangling surfaces, the amount of distillable entanglement is mostly largest for free theories, while counterexample exists for free Dirac fermion in three dimensions. For holographic CFTs described by higher derivative gravity, χ may increase or decrease, depending on the sign of the relevant parameters. Our results may reveal a more profound connection between geometry and distillable entanglement.

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1. Introduction

Entanglement may be the most mysterious phenomenon inherently related to quantum mechanics. Roughly speaking entanglement describes the presence of correlations, while the nature of entanglement can be either classical or quantum. So given a quantum state, a central question is to distinguish and quantify quantum entanglement from the classical counterpart. By far it has been realised that at least for pure states, the nature of entanglement can be completely characterised by the well-known Bell/Clauser–Horne–Shimony–Holt (CHSH) inequalities, but the case of mixed states is much less clear.

Let us focus on a pure state ρ of a bipartite system, whose Hilbert spaces are denoted $\mathcal{H}_L$ and $\mathcal{H}_R$. If ρ can be expressed as

$$\rho = \sum_i p_i \rho_i^L \otimes \rho_i^R, \quad \sum_i p_i = 1, \quad p_i \geq 0, \tag{1.1}$$

then ρ is separable, otherwise it is entangled. Since separable states can be produced using only local operations and classical communications (LOCC), they are classically correlated. However, for mixed states the situation becomes much more complex, for example, the intuition that the only states that satisfy the Bell inequalities are the separable ones fails for mixed states. Several entanglement measures have been proposed because of the intricate nature of mixed state entanglement, among which a computable one is the entanglement negativity (EN) [1].

For a bipartite system, the entanglement negativity is determined by the absolute values of the partial transposed density matrix (a detailed definition will be given in the next section). Such a concept can also be generalised in the framework of relativistic quantum field theories [2,4,5], where computations of entanglement negativity in $1+1$-dimensional QFTs amount to evaluating twist operator correlations in [4,5]. Furthermore, progress in recent years has enabled us to 'geometrise' the entanglement entropy (EE) in the context of gauge/gravity duality [6,7]. Such progress has opened up new windows towards a deeper understanding on the connections between geometry and entanglement.

It is therefore natural to ask to which extent we can extract new properties of EN of general d-dimensional conformal field theories in the framework of holography. In [8] it was pointed out that for an entangling region $\mathcal{A}$, the EN in a pure state possesses the following properties:

- The leading divergent term scales as the area of the entangling surface $\partial\mathcal{A}$, which is analogous to the case of EE;
- The sub-leading divergent terms have an identical structure to that in the EE for the reduced density matrix $\rho_{\mathcal{A}}$;
- The value of the negativity generally takes a larger value than the corresponding EE, whose difference was conjectured to be in a geometric factor.

In particular, the authors of [8] defined the following quantity,

$$\chi = \left| \frac{C^{\text{univ}}[\mathcal{E}_N]}{C^{\text{univ}}[S_{\text{EE}}]} \right|, \tag{1.2}$$

where $C^{\text{univ}}[\cdot\cdot]$ denotes the universal part of the entanglement negativity $\mathcal{E}_N$ and the EE S_{EE}. It was claimed in [8] that χ gives a precise measure of the entanglement negativity for the ground state in terms of the EE. In other words, the difference between the negativity and the EE may be encoded in χ and χ should just depend on the geometry of the entangling surface $\partial\mathcal{A}$. The values