Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 910 (2016) 665–684

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

The ground state of the $D = 11$ supermembrane and matrix models on compact regions

Lyonell Boulton [a,*], Maria Pilar Garcia del Moral [b], Alvaro Restuccia [b,c]

[a] *Department of Mathematics & Maxwell Institute for the Mathematical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom*

[b] *Departamento de Física, Universidad de Antofagasta, Avda Universidad de Antofagasta 02800, Antofagasta, Chile*

[c] *Departamento de Física, Universidad Simón Bolívar, Valle de Sartenejas, 1080-A Caracas, Venezuela*

Received 13 May 2016; accepted 21 July 2016

Available online 28 July 2016

Editor: Stephan Stieberger

Abstract

We establish a general framework for the analysis of boundary value problems of matrix models at zero energy on compact regions. We derive existence and uniqueness of ground state wavefunctions for the mass operator of the $D = 11$ regularized supermembrane theory, that is the $\mathcal{N} = 16$ supersymmetric $SU(N)$ matrix model, on balls of finite radius. Our results rely on the structure of the associated Dirichlet form and a factorization in terms of the supersymmetric charges. They also rely on the polynomial structure of the potential and various other supersymmetric properties of the system.

1. Introduction

Physical theories subject to boundary conditions play a crucial role in the study of physical properties at high energies and they are also notably relevant in the study of some condensed

* Corresponding author.

E-mail addresses: l.boulton@hw.ac.uk (L. Boulton), maria.garciadelmoral@uantof.cl (M.P. Garcia del Moral), arestu@usb.ve (A. Restuccia).

URLs: http://www.ma.hw.ac.uk/~lyonell/ (L. Boulton), http://faciba.uantof.cl/index.php/ (M.P. Garcia del Moral), http://faciba.uantof.cl/index.php/ (A. Restuccia).

http://dx.doi.org/10.1016/j.nuclphysb.2016.07.025

matter effects. Recently, supersymmetric boundary conditions have received renewed attention, due to their connection with quantum phase transition at the boundary of topological superconductors. We refer to [1] and [2], where a systematic analysis of a class of $N = 2$ supersymmetric theories subject to several boundary conditions was carried out.

Matrix models related to the regularization of field theories with boundary conditions have been studied in order to test the AdS/CFT conjecture at finite temperature. In part this is due to its relation in the bulk picture with the black hole microstates in this regime [3]. An analysis of the unbounded matrix model wavefunction, related to the $(0 + 1)$ supersymmetric Yang–Mills ground state, has been considered in various other cases [4–7]. In the context of M-theory and for the regularized supermembrane, this has been studied in [8] and subsequent work. For a matrix model wavefunction perspective see [9–14], for different aspects of Lorentz invariance see [15,16], for inner solutions see [17] and for asymptotic solutions see [18–22].

M-theory seen as a unification theory should provide a quantum description of $D = 11$ supergravity. In this setting, String Theory is regarded as a perturbative limit of M-theory which should include all non-perturbative effects. The $D = 11$ supermembrane describes relevant degrees of freedom of M-theory, because it couples consistently to a $D = 11$ supergravity background without destroying the local fermionic symmetry [23]. This provides strong evidence that the ground state of the $D = 11$ supermembrane should correspond to a wavefunction constructed in terms of the $D = 11$ supergravity multiplet.

In spite of several insightful attempts [8–22], a construction of the ground state wavefunction of the $D = 11$ supermembrane has remained an elusive challenge since the original analysis performed in [8]. It is well-known that the Hamiltonian of the theory formulated on a Minkowski spacetime in the Light Cone Gauge [8] is the sum of two components. One of the components is associated with the kinematics of the center of mass of the supermembrane described in terms of the zero modes. The other component is the mass operator of the supermembrane which only depends on the non-zero modes.

The ground state wavefunction, Ψ, factorizes into two parts

$$\Psi = \Psi^0 \Psi^{\text{non-zero}}.$$

The zero mode wavefunction, Ψ^0, is responsible for the planar wave associated to the supergravity supermultiplet. The non-zero mode wavefunction, $\Psi^{\text{non-zero}}$, should be annihilated by the mass operator of the supermembrane and should be a singlet under $SO(9)$. This ensures that the full wavefunction Ψ is the unique solution constructed in terms of the $D = 11$ supergravity multiplet.

Rigourous treatments of the spectrum (in particular the ground state of the supermembrane) have been achieved by means of an $SU(N)$ regularization of the theory [8,24–26]. These always involve the quantum mechanics of an $SU(N)$ matrix model which was originally introduced in a different context in [4,27,28]. The corresponding Hamiltonian is the starting point of the matrix model theory developed in [29].

The regularized $D = 11$ supermembrane was rigorously shown to have a continuous spectrum, the segment from zero to infinity, in [30]. The compactification in a sector of the target space by itself does not change this property [31]. However, the spectrum becomes purely discrete with finite multiplicities, when the maps describing the regularized Hamiltonian satisfy a topological condition [32] corresponding to a non-trivial central charge in the supersymmetric algebra. See [33–38] for a rigourous treatment in this respect. The setting developed in [38] also shows that the BMN matrix model [39] has purely discrete spectrum when considered beyond its semi-classical limit. This argument extended the semiclassical analysis performed in [39].