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Convexity at finite temperature and non-extensive thermodynamics

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Abstract

Assuming that tunnel effect between two degenerate bare minima occurs, in a scalar field theory at finite volume, this article studies the consequences for the effective potential, to all loop orders. Convexity is achieved only if the two bare minima are taken into account in the path integral, and a new derivation of the effective potential is given, in the large volume limit. The effective potential then has a universal form, it is suppressed by the space time volume, and does not feature spontaneous symmetry breaking as long as the volume is finite. The finite temperature analysis leads to surprising thermal properties, following from the non-extensive expression for the free energy. Although the physical relevance of these results is not clear, the potential application to ultra-light scalar particles is discussed.

1. Introduction

For a scalar theory with several degenerate vacua, it is usually assumed that spontaneous symmetry breaking (SSB) occurs and that one specific vacuum is chosen. This is actually true for infinite volume, where the tunnel effect between different vacua is completely suppressed. But for finite volume, even the slightest tunneling possibility between different degenerate vacua should allow these to play an equivalent role at equilibrium, for the true vacuum of the dressed theory to be a superposition of the bare vacua. It has been known for a long time that the

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effective potential is then convex [1], which is a consequence of its definition in terms of a Legendre transform [2]. It has been shown, from the early days of effective potential methods [3], that convexity cannot be achieved when quantisation is based on one vacuum only, if the bare potential has several degenerate vacuua [4]. The effective potential actually becomes convex as a consequence of the competition of the different non-trivial saddle points [5], and is thus a non-perturbative effect.

Gauge fixing could also impose a specific vacuum for the scalar field, therefore avoiding convexity of the effective potential. Nevertheless, a construction of a convex effective Higgs potential is given in [6], where gauge fixing picks two points on the manifold of vacuua of the bare potential. It is shown that a convex effective potential can then be obtained from a linear interpolation between these two vacuua, to any loop order. The explicit form of the effective potential is not given though.

An explicit construction of the convex saddle point effective potential (ignoring loop corrections) is given for the first time in [7], where the effective potential is derived as an expansion in the classical field, up to the fourth order, for a finite spacetime volume $V^{(4)}$. In this work, the degenerate vacuua of an $O(N)$-symmetric scalar theory, with $|\vec{\phi}_{vac}| = v$, all contribute to the saddle point approximation for the partition function. The path integral quantisation is then followed step by step, where all the quantities are expanded in either the source or the classical field. The resulting effective potential is a convex polynomial, which is suppressed by $V^{(4)}$, as a consequence of an interpolation between the different bare vacuua. Therefore it becomes flat in the limit of infinite volume, and SSB is reached only in this limit, where the true vacuum is an arbitrary point of a flat N-ball with radius v.

Although not studied in [7], Goldstone modes then arise, which should stay massless to all orders in perturbation theory. This is shown in [8], using an improved Conwall–Jackiw–Tomboulis effective action [9].

The present article (restricted to a single real scalar field) shows an alternative construction, which is not based on an expansion in fields, but in $(v^4 V^{(4)})^{-1}$ instead. This approach leads to an effective potential which is valid to all orders in the classical field, and whose Taylor expansion to the fourth order is consistent with [7]. The true vacuum of the (dressed) theory is located at $\phi = 0$, and SSB does not occur as long as $V^{(4)}$ is finite. This result is first obtained in the saddle point approximation for the partition function, at zero temperature, and we show that one-loop corrections do not change the functional form of the effective potential, but only redefine the mass scale v. We show then that these results hold at finite temperature too, as long as one is below a critical temperature.

Concerning the true vacuum of a theory, we note that a systematic construction of effective theories is done in [10], where tadpoles are removed consistently with the true vacuum. In the situation where the bare vacuua are not degenerate, one can also consider the famous problem of false vacuum decay [11], for which radiative corrections are considered in [12], and gauge invariance is shown in [13]. But these studies assume a time dependence of the ground state of the theory, whereas we consider here the equilibrium situation in the case of a symmetric potential.

This article is structured as follows. The explicit construction of the effective potential at zero temperature, within the saddle point approximation, is done in section 2. The one-loop corrections are calculated in section 3, and the results are expected to be identical at higher order loops, up to a redefinition of the mass scale v of the theory. The extension to finite temperature is done in section 4, where the effective potential is suppressed by the three-dimensional space volume, as long as one stays below the usual critical temperature. This suppression holds in an interval