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Fractional Galilean symmetries

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Abstract

We generalize the differential representation of the operators of the Galilean algebras to include fractional derivatives. As a result a whole new class of scale invariant Galilean algebras are obtained. The first member of this class has dynamical index $z = 2$ similar to the Schrödinger algebra. The second member of the class has dynamical index $z = 3/2$, which happens to be the dynamical index Kardar–Parisi–Zhang equation.

1. Introduction

Locality has been the guiding principal of many branches of physics. This has come about in opposition to the concept of action at a distance. That is to say for something to influence another thing an entity such as a field or particle must travel between the two points. However phenomena such as the EPR experiments point to the plausibility of non-local theories in quantum mechanics [1]. Another setting in which non-local theories arise is equilibrium thermodynamics, where systems have had enough time to reach a state of equilibrium where long range correlations have come into existence. An example of recent work in this direction is given by [2], where conformal invariance of long range Ising model is discussed. This means that non-local theories may

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be acceptable. If locality is given up a very rich arena of possible models in theoretical physics arises, one of which is the option of having fractional derivatives. Consequently this opens up the field for seeking symmetry algebras which are composed of derivatives with fractional order. In this paper we are going to see what we can get if Galilean symmetry is augmented with fractional derivative operators. We find a whole new class of scale and Galilean invariant symmetries, with previously unknown dynamic exponents. Galilean symmetries have received some attention recently due to their appearance in the AdS/CFT correspondence [3–11]. The Galilean symmetries are composed of translations in time and space plus rotations in space and boosts to inertial frames that move with constant velocity. One can add to this set, scale transformations and boosts to frames of reference which conserve higher derivatives such as accelerations plus "toroidal" transformations of the time component. These symmetries lead to L-Galilei algebras [12,13] where L is an integer, for related works see [14–16]. The disappointment in L-Galilean symmetries is that not all dynamical exponents "z" arise, although the first two members of this class i.e. Galilean Conformal Algebra (GCA) $z = 1$ [17–19] and Schrödinger symmetry $z = 2$, [20–25] are well known. Attempts to obtain other dynamical exponents exist [26] where the dynamical index of $z = \frac{3}{2}$ was obtained. However this remained a singular case. Other cases have been suggested by Henkel and Stoimenov in which further values of dynamical index are obtained using the age subalgebra of the Schrödinger algebra [27,28]. However the algebra only closes on the solution space. Besides the exponent, the geometrical meaning of infinitesimal fractional algebra has been analyzed in the context of the Schrödinger algebra in [28].

In this paper we show that admitting fractional derivatives allows a new class of Galilean symmetries, which we call F-Galilean where F is an integer and the dynamical exponent is given by

$$z = \frac{F+1}{F} \tag{1.1}$$

Thus for $F = 1$, we have the well-know exponent $z = 2$ (this symmetry is smaller than Schrödinger symmetry) and for $F = 2$ we get $z = \frac{3}{2}$, i.e. the well-known exponent of the Kardar–Parisi–Zhang (KPZ) equation [29]. The price we paid for this extension is non-locality through introduction of fractional derivatives. This paper is organized as follows. In section 2 we give a brief introduction to L-Galilei algebras, which in fact encompasses all known galilean algebras. In Section 3 we introduce fractional derivatives and some of their properties. In section 4 we derive the new class of galilean algebras which posses differential operator representations including fractional derivatives. It is in fact through the consistency of these representations that we derive this new class of non-local galilean symmetries.

2. L-Galilei algebra

L-Galilei algebra were introduced independently by Henkel [12] and Negro, del Olmo and Rodríguez-Marco [13]. Here we briefly review their method. First, let us recall the two well-known non-relativistic symmetries: Schrödinger and conformal Galilean algebras.

– Schrödinger algebra

Schrödinger algebra consists of the following generators. The center is formed by the generators of the Galilean transformations

$$P = -\partial_x, \qquad H = -\partial_t, \qquad B = -t\partial_x \tag{2.1}$$

which guarantees invariance under translations in space and time, plus Galilean boost.