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Exact solution of the trigonometric $SU(3)$ spin chain with generic off-diagonal boundary reflections

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Abstract

The nested off-diagonal Bethe ansatz is generalized to study the quantum spin chain associated with the $SU_q(3)$ R-matrix and generic integrable non-diagonal boundary conditions. By using the fusion technique, certain closed operator identities among the fused transfer matrices at the inhomogeneous points are derived. The corresponding asymptotic behaviors of the transfer matrices and their values at some special points are given in detail. Based on the functional analysis, a nested inhomogeneous $T-Q$ relations and Bethe ansatz equations of the system are obtained. These results can be naturally generalized to cases related to the $SU_q(n)$ algebra.

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1. Introduction

Exact solution is a very important issue in studies of statistical mechanics, condensed matter physics, quantum field theory and mathematical physics [1,2] since those results can provide important benchmarks for understanding physical effects in a variety of systems. The coordinate Bethe ansatz and the algebraic Bethe ansatz are two powerful methods to obtain the exact solution of the integrable systems [3–7]. With these methods, many interesting exactly solvable models, such as the one-dimensional Hubbard model, supersymmetric $t-J$ model, Heisenberg spin chain and the δ-potential quantum gas model, were exactly solved. For integrable systems with $U(1)$ symmetry, it is easy to find a reference state and these conventional Bethe ansatz can be applied to. Indeed, most of the previous studies focus on periodic or diagonal open boundary conditions without breaking the $U(1)$ symmetry. However, there exists another kind of integrable systems which does not have the $U(1)$ symmetry, such as the integrable systems with generic off-diagonal boundary reflections. Because the reference state of this kind of integrable system is absent, the conventional Bethe ansatz methods are failed. On the other hand, many interesting phenomena arise in this kind of systems, such as the topological elementary excitations in the spin-1/2 torus [8], spiral phase in the Heisenberg model with unparallel boundary magnetic field [9] and stochastic process in non-equilibrium statistical mechanics [10–12]. Motivated by these important applications, many interesting methods such as the q-Onsager algebra [13–15], the modified algebraic Bethe ansatz [16–19] and the Sklyanin's separation of variables (SoV) [20–24] were also applied to some integrable models without $U(1)$ symmetry. Other interesting progress can be found in [25–29].

Recently, a new approach, i.e., the off-diagonal Bethe ansatz (ODBA) [8] was proposed to obtain exact solutions of generic integrable models either with or without $U(1)$ symmetry. Several long-standing problems were then solved [30–36] via this method. For comprehensive introduction to this method we refer the readers to [37]. In order to study the high rank integrable models, the nested version of ODBA has been proposed for the isotropic (or rational) models [33]. In this paper, we study the anisotropic rank-2 spin model with generic integrable boundary conditions. Here the R-matrix is the trigonometric one associated with the $SU_q(3)$ algebra and the boundary reflection matrices are the most generic reflection matrices which have non-vanishing off-diagonal elements. Because the off-diagonal elements of the reflection matrices break the $U(1)$ symmetry, the exact solution of the system has been missing even its integrability was known for many years ago. By using the fusion technique and nested ODBA, we successfully obtain the closed operator identities, the values at the special points and the asymptotic behaviors. Based on them, we construct the nested inhomogeneous $T-Q$ relation and obtain the eigenvalue of the transfer matrix thus the energy spectrum of the system. These results can be generalized to multiple components spin chains related to more higher rank algebra cases.

The paper is organized as follows. Section 2 is the general description of the model. The $SU_q(3)$ R-matrix and corresponding generic integral non-diagonal boundary reflection matrices are introduced. In Section 3, by using the fusion technique, we derive the closed operator identities for the fused transfer matrices and the quantum determinant. The asymptotic behaviors of the fused transfer matrix and their values at special points are also obtained. In Section 4, we list some necessary functional relations which are used to determine the eigenvalues. Section 5 is devoted to the construction of the nested inhomogeneous $T-Q$ relation and the Bethe ansatz equations. In Section 6, we summarize our results and give some discussions. Some results related to the other types of the general off-diagonal boundary reflections are given in Appendix A.