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Boundary effects in entanglement entropy

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Abstract

We present a number of explicit calculations of Renyi and entanglement entropies in situations where the entangling surface intersects the boundary of d-dimensional Minkowski spacetime. When the boundary is a single plane we compute the contribution to the entropy due to this intersection, first in the case of the Neumann and Dirichlet boundary conditions, and then in the case of a generic Robin type boundary condition. The flow in the boundary coupling between the Neumann and Dirichlet phases is analyzed in arbitrary dimension d and is shown to be monotonic, the peculiarity of $d = 3$ case is noted. We argue that the translational symmetry along the entangling surface is broken due the presence of the boundary which reveals that the entanglement is not homogeneous. In order to characterize this quantitatively, we introduce a density of entanglement entropy and compute it explicitly. This quantity clearly indicates that the entanglement is maximal near the boundary. We then consider the situation where the boundary is composed of two parallel planes at a finite separation and compute the entanglement entropy as well as its density in this case. The complete contribution to entanglement entropy due to the boundaries is shown not to depend on the distance between the planes and is simply twice the entropy in the case of single plane boundary. Additionally, we find how the area law, the part in the entropy proportional to the area of entire entangling surface, depends on the size of the separation between the two boundaries. The latter is shown to appear in the UV finite part of the entropy.

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1. Introduction

Entanglement entropy is a useful tool which plays an important role in modern physics. First introduced [1] in order to explain the black hole entropy, it was later shown to be very efficient in measuring the quantum entanglement between sub-systems separated by a surface. In infinite spacetime this surface is necessarily compact so that it divides the spacetime into two complementary regions. The correlations present in the quantum system across the entangling surface produce the non-trivial entropy which is essentially determined by the geometry of the surface. The geometrical nature of entanglement entropy explains why it finds so many applications in various fields of physics, from black holes and holography to integrable models and quantum computers [2]. For some recent progress in measuring entanglement entropy see [3].

For conformal field theories, the entanglement entropy plays a special and important role since the logarithmic terms in the entropy are related to the conformal anomalies, as suggested in [4]. In infinite spacetime, the anomaly appears only in even dimensions. In parallel, for compact entangling surfaces, only in even dimensions there appear the logarithmic terms in the entropy.

Recently there has been some progress in understanding the conformal anomalies in the case where the spacetime is not infinite but has some boundaries, [5–8] (for earlier works see [10]). It is interesting that in the presence of boundaries the integrated anomaly is non-vanishing in odd spacetime dimensions, the relevant contribution being produced by the boundary terms only, [7]. Thus, it becomes an interesting and urgent problem to understand the precise structure of the entropy for entangling surface which intersects the boundary of a spacetime. In the holographic context, this and related problems were studied in [11,12], and on the field theory side in [13]. The precise calculation for free fields of various spin in dimension $d = 3$ has been done in [9] where it was shown that the logarithmic term in the entropy in this case is proportional to the number of intersections the entangling surface has with the boundaries. In higher dimensions it was suggested that, unlike the case of compact closed surfaces, the logarithmic terms in the entropy of a surface intersecting the boundary are present in any, odd and even, dimensions.

The boundary phenomenon in entanglement entropy is certainly more general and is not restricted only to conformal field theories, for earlier works see [13,14]. Yet, the explicit calculations for arbitrary boundaries and surfaces are technically complicated, if even possible. Therefore, we find it instructive to first analyze the problem in some simple cases, where the spacetime is flat and the boundary is composed by a collection of planes. In this paper we present a number of explicit calculations, for a free massive scalar field, of entanglement entropy in the case where the entangling surface is a plane which crosses orthogonally the boundary. The main focus is made on the role of the boundary conditions. The latter can be viewed as some form of boundary interactions. The general Robin type condition then interpolates between the Neumann condition in the weak coupling regime and the Dirichlet condition in the strong coupling regime. We study the respective behavior of entanglement entropy when the boundary coupling passes between these two regimes.

The paper is organized as follows. In Section 2 we review the standard replica method that uses the heat kernel and the conical singularity technology. We demonstrate how this method works for a simple case of infinite plane in infinite (without boundaries) Minkowski spacetime. This technology is then applied in Section 3 to the case of a single plane boundary with the Neumann (Dirichlet) boundary condition. The case of a general Robin type condition is considered in Section 4. We observe some inequalities for the entropy for different boundary conditions in Section 5. The monotonicity of the entropy with respect to the boundary coupling is demonstrated in Section 6. Two parallel boundaries and the effects of the finite size are considered in Section 7.