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Generalized planar black holes and the holography of hydrodynamic shear

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Abstract

AdS black holes with planar event horizon topology play a central role in AdS/CFT holography, and particularly in its applications. Generalizations of the known planar black holes can be found by considering the Plebański–Demiański metrics, a very general family of exactly specified solutions of the Einstein equations. These generalized planar black holes may be useful in applications. We give a concrete example of this in the context of the holographic description of the Quark–Gluon Plasma (QGP). We argue that our generalized planar black holes allow us to construct a model of the internal shearing motion generated when the QGP is produced in peripheral heavy-ion collisions. When embedded in string theory, the bulk physics is in fact unstable. We find however that this instability may develop too slowly to affect the evolution of the plasma, except possibly for high values of the quark chemical potential, such as will be studied in experimental scans of the quark matter phase diagram.

1. Holographic description of the internal motion of the QGP

Collisions of heavy ions [1] are believed to produce a state of matter known as the Quark–Gluon Plasma or QGP. One theoretical approach to understanding this state is based on *holography*, in which the QGP is modeled by a field theory dual to a gravitational system, a thermal AdS black hole [2–5]. Because the QGP exists in Minkowski space, one needs to use black holes with topologically planar event horizons; fortunately, these do exist in the asymptotically AdS

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case [6], though, as we shall see, these objects differ, in many important particulars, from their counterparts with topologically spherical sections.

The QGP, as produced in collisions, is far from being a static system: most obviously, it expands very rapidly, and much work is currently devoted to finding a dual description of the expanding QGP (see [5,7–10] and references therein). However, it has recently come to be appreciated that, in non-central ("peripheral") collisions, the QGP is also subjected to an extremely intense *shearing* motion, because of the transverse non-uniformity of the colliding nuclei [11–13]. The large vorticity thus generated [14–18] may prove to have observable consequences, through quark polarization, and it may be important for forthcoming attempts to simulate peripheral collisions using holography [19].

Clearly it is important to determine what the holographic approach has to teach us about this shearing effect. Since the shearing imparts a very large angular momentum to the plasma, the natural suggestion [20] is to consider AdS–Kerr [21,22] black holes in the bulk. However, while the AdS–Kerr black hole (with a topologically spherical event horizon) does induce rotation at infinity, the angular velocity is constant (that is, independent of direction) there, and hence there is no shear. This spacetime is indeed of interest for possible holographic descriptions of systems which are genuinely *rotating* (see [23] for an explicit example, and [24] for another potential application), but this is not what we seek here.

In any case, the AdS–Kerr metric does not have a conformal boundary which is globally conformal to Minkowski spacetime, so one should really begin rather with non-static black holes which do have that property — that is, with planar black holes possessing angular momentum. Such black holes were discovered by Klemm, Moretti and Vanzo [25], and one finds that, in addition to inducing a more reasonable geometry at infinity, these "KMV_0 spacetimes" bring with them a major advantage: the angular velocity at infinity depends on one of the spatial coordinates, *thus inducing an effective shearing motion on the boundary*. The KMV_0 spacetimes, and their electrically charged ("$QKMV_0$") versions, are therefore prime candidates for building a holographic description of the shearing QGP. Such a description may be very important, because it often happens that certain effects are apparent on one side of a holographic duality, but obscure on the other. Work in this direction was described in [26,27].

The shearing motion of a fluid is described by a *velocity profile*, an expression of the (dimensionless) velocity $v(x)$ as a function of transverse distance from some axis. (In the case of plasma generated by heavy-ion collisions, it is customary to choose the axis to be that of the collision, that is, the axis along which the velocity vanishes; it is conventional to take it to be the z axis.) This function is of basic importance, since it describes the internal dynamics of the plasma. Unfortunately, in the extreme conditions in the aftermath of a heavy-ion collision, it is difficult to predict the *precise* shape $v(x)$ actually takes; however, typical shapes arising in shear flows are known (see [28], p. 133). Such a shape, chosen because it is consistent with causality, is shown in Fig. 1.

Crucially, some aspects of fluid behavior actually depend only on the *general shape* of the profile, not on the details; these aspects, therefore, can be discussed in the QGP case, even in the absence of a precisely specified $v(x)$ function. Most important among these is the question as to whether the shearing motion described by $v(x)$ is stable, in the hydrodynamic sense. Frequently it is not, the most well-known example being the *Kelvin–Helmholtz instability*, which is important in numerous fluid mechanics problems where shear causes a laminar flow to become turbulent. Remarkably, all one needs to detect this instability, in the case of a monotone profile, is a knowledge of the *sign* of the second derivative of $v(x)$. To be precise, a necessary (and usually