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Compton scattering: From deeply virtual to quasi-real

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Abstract

We address the question of interpolation of the virtual Compton scattering process off a polarized nucleon target between the deeply virtual regime for the initial-state photon and its near on-shell kinematics making use of the photon helicity-dependent Compton Form Factors (CFFs) as a main ingredient of the formalism. The five-fold differential cross section[1] for the reaction with all possible polarization options for the lepton and nucleon spins is evaluated in terms of CFFs in the rest reference frame of the initial-state nucleon. We suggest a rather simple parametrization of the Compton hadronic tensor in terms of CFFs which are free from kinematical singularities and are directly related, at large photon virtualities, to generalized parton distributions. We also provide a relation of our basis spanned by a minimal number of Dirac bilinears to the one introduced by Tarrach for the parametrization of the virtual Compton tensor and utilize the former to establish a set of equalities among our CFFs and generalized polarizabilities. As a complementary result, we express Compton scattering in the Born approximation in terms of CFFs as well.

Keywords: Compton scattering; Compton form factors; Generalized parton distributions; Generalized polarizabilities

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1. Introduction

Virtual Compton scattering on a nucleon, $\gamma^*(q_1)N(p_1) \to \gamma(q_2)N(p_2)$, plays a distinguished role in the quest to access its internal content and unravel the mysteries of strong interactions. The reason for this is multifold. Experimentally, the scattering process off a proton can be measured in a straightforward fashion, free of complications of composite probes, via scattering of leptons on

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[1] Cross section formulae are available in a Mathematica code upon request, contact dieter.mueller@tp2.rub.de.

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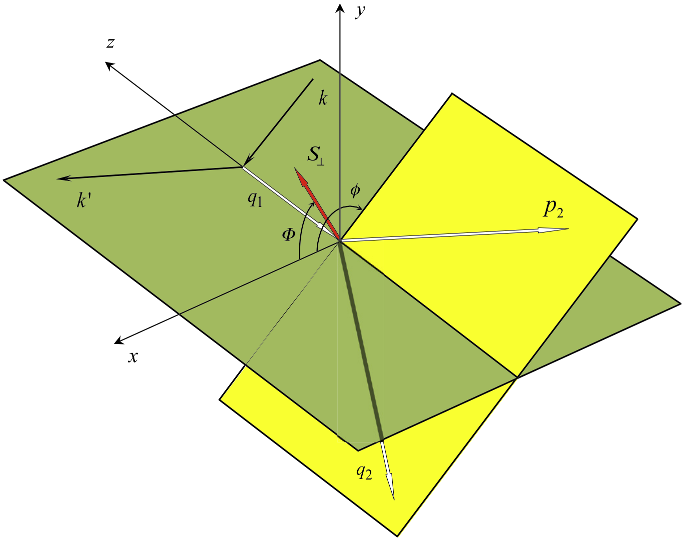

Fig. 1. The target rest frame, used in this work, is the same as adopted in our previous consideration [1]. The z-axis is directed counter-along the photon three-momentum $\boldsymbol{q}_1$, the x-component of the incoming electron momentum $\boldsymbol{k}$ is chosen to be positive. The angles parametrizing the five-fold cross section (1) are defined as follows: ϕ is the azimuthal angle between the lepton plane and the recoiled proton momentum, while the difference $\varphi \equiv \Phi - \phi$ for fixed ϕ is determined by the direction of the transverse nucleon polarization vector component $\boldsymbol{S}_\perp = (\cos\Phi, \sin\Phi)$.

a hydrogen target. The five-fold differential cross section for the emission of an on-shell photon to the final state, $\ell(k)N(p_1) \to \ell(k')N(p_2)\gamma(q_2)$, reads

$$d\sigma = \frac{\alpha_{\rm em}^3 x_{\rm B} y^2}{16\pi^2 \mathcal{Q}^4 \sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2 dx_{\rm B}\, d\mathcal{Q}^2\, d|t|\, d\phi\, d\varphi, \tag{1}$$

in the approximation that neglects the mass of the lepton. The phase space is parametrized by the Bjorken variable $x_{\rm B} = \mathcal{Q}^2/(2p_1 \cdot q_1)$, which is in turn determined by the momentum $q_1 = k - k'$ of the initial-state photon of virtuality $\mathcal{Q}^2 = -q_1^2$, the square of the t-channel momentum $t = (p_2 - p_1)^2$, the azimuthal angle ϕ of the recoiled nucleon, and for a transversally polarized target yet another (relative) angle φ, where the latter two are defined in the rest frame of the target as depicted in Fig. 1. Finally, we introduce the variable $y = p_1 \cdot q_1 / p_1 \cdot k$ for the lepton energy loss and a shorthand notation for $\epsilon = 2x_{\rm B} M/\mathcal{Q}$ that incorporates nonvanishing target mass effects. In the above five-fold cross section, the leptoproduction amplitude $\mathcal{T}$ is a linear superposition of the Bethe–Heitler (BH) and virtual Compton scattering (VCS) amplitudes, depending on whether the real photon is emitted off the lepton or nucleon, respectively. In the scattering amplitude

$$\mathcal{T} = \mathcal{T}^{\rm BH} + \mathcal{T}^{\rm VCS}, \tag{2}$$

the former is determined in terms of the nucleon matrix element of the quark electromagnetic current j_μ

$$J_\mu = \langle p_2 | j_\mu(0) | p_1 \rangle, \tag{3}$$

while the hadronic Compton tensor,

$$T_{\mu\nu} = i \int d^4 z\, {\rm e}^{\frac{i}{2}(q_1+q_2)\cdot z} \langle p_2 | T\left\{ j_\mu(z/2) j_\nu(-z/2) \right\} | p_1 \rangle, \tag{4}$$