Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 878 [PM] (2014) 117–149

www.elsevier.com/locate/nuclphysb

Existence of non-Abelian vortices with product gauge groups

Xiaosen Han [a,b,*], Chang-Shou Lin [b]

[a] *Institute of Contemporary Mathematics, School of Mathematics, Henan University, Kaifeng, Henan 475004, PR China*
[b] *Taida Institute for Mathematical Sciences, Center for Advanced Study in Theoretical Science, National Taiwan University, Taipei, 10617, Taiwan*

Received 22 October 2013; accepted 13 November 2013

Available online 21 November 2013

Abstract

In this paper we establish several sharp existence and uniqueness theorems for some non-Abelian vortex models arising in supersymmetric gauge field theories. We prove these results by studying a family of systems of elliptic equations with exponential nonlinear terms in both doubly periodic-domain and planar cases. In the doubly periodic-domain case we obtain some necessary and sufficient conditions, each explicitly expressed in terms of a single inequality interestingly relating the vortex numbers, to coupling parameters and size of the domain, for the existence of solutions to these systems. In the planar case we establish the existence results for any vortex numbers and coupling parameters. Sharp decay estimates for the planar solutions are also obtained. Furthermore, the solutions are unique, which give rise to the quantized integrals in all cases.

Keywords: Non-Abelian vortices; Supersymmetric gauge theories; BPS equations; Existence; Uniqueness

1. Introduction

Vortices are important objects in various branches of physics [36] including condensed matter physics [1,28], particle physics [25], string theory and cosmology [23,29,48]. It is well

Funded by SCOAP[3].

* Corresponding author at: Institute of Contemporary Mathematics, School of Mathematics, Henan University, Kaifeng, Henan 475004, PR China. Tel./fax: +86 371 23881696.
E-mail address: xiaosenhan@gmail.com (X. Han).

0550-3213
http://dx.doi.org/10.1016/j.nuclphysb.2013.11.009

known there admit the Abrikosov–Nielsen–Olesen vortices [1,37] for the classical Abelian Higgs model, whose static limit is also known as the Ginzburg–Landau model for superconductivity [17]. The first rigorous existence result for vortex configurations were established by Taubes [45,46] for the Ginzburg–Landau model [17]. Since then various analytic methods for studying the existence of vortices and other topological solitons have been developed [28,44,50].

During the last ten years much attention has been concentrated to vortices in non-Abelian gauge field theories since they are related to the fundamental puzzle in theoretical physics, quark confinement or color confinement [18,42]. In fact, in their famous work [38] Seiberg and Witten use non-Abelian color charged monopoles and vortices to interpret quark confinement. Motivated by the importance of non-Abelian vortices in the understanding of monopole and quark confinement, a wide class of non-Abelian gauge theories were developed in [3,21, 22,39]. See [4,7–12,15,31,40,43] for more recent progress and [14,30,41,47] for surveys on this topic. In these theories there arise many interesting and challenging systems of elliptic partial differential equations. It is interesting to carry out a rigorous analysis for these partial differential equations from both physical and mathematical points of view. In this respect, we cite the work [6,32–35], where a series of existence and uniqueness results were established.

The purpose of this paper is to establish sharp existence theories for the non-Abelian vortex model with product moduli proposed in [7], and for the Yang–Mills–Higgs model with gauge groups $U(1) \times SO(2M)$ and $U(1) \times SU(N)$ in [11,12,19]. We recall that, by the approach of moduli matrix [13,26], a series of systems of non-Abelian BPS vortex equations were obtained in [7,11,12,19]. For each of these systems we establish sharp existence, uniqueness, asymptotic behavior and quantized integral results. In particular, for these models over doubly periodic domain we obtain some necessary and sufficient condition, each explicitly expressed in a single inequality interestingly relating the vortex numbers, to coupling parameters and size of the domain, for the existence of solutions. Over the full plane, we obtain existence and uniqueness results for any vortex numbers and coupling parameters. Furthermore, the explicit decay estimates for planar solutions are established. Our approach is based on the direct minimization methods recently developed in [32,33].

The rest of our paper is organized as follows. In Section 2 we review a system of vortex equations from the model proposed in [7] and state a sharp existence result Theorem 2.1 for this problem over a doubly periodic-domain and the full plane. In Sections 3 and 4 we prove Theorem 2.1 for the doubly periodic-domain and planar cases, respectively. Sections 5 and 6 are devoted to establishing existence results for the BPS vortex equations [11,19] arising in the Yang–Mills–Higgs model with gauge groups $U(1) \times SO(2M)$ and $U(1) \times SU(N)$ separately.

2. Non-Abelian vortex model with product gauge group

In this section we consider the non-Abelian vortex model proposed in [7] with gauge group G, where

$$G = \frac{U(1) \times SU(n) \times SU(r)}{\mathbb{Z}_k},$$

and k is the least common multiple of n and r. Following [7], the action density of the model reads as