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Leptonic unitarity triangles and effective mass triangles of the Majorana neutrinos

Zhi-zhong Xing [a,b,c], Jing-yu Zhu [a,*]

[a] *Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[b] *School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*
[c] *Center for High Energy Physics, Peking University, Beijing 100080, China*

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Abstract

Given the best-fit results of six neutrino oscillation parameters, we plot the Dirac and Majorana unitarity triangles (UTs) of the 3×3 lepton flavor mixing matrix to show their real shapes in the complex plane. The connections of the three Majorana UTs with neutrino–antineutrino oscillations and neutrino decays are explored, and the possibilities of right or isosceles UTs are discussed. In the neutrino mass limit of $m_1 \to 0$ or $m_3 \to 0$, which is definitely allowed by current data, we show how the six triangles formed by the effective Majorana neutrino masses $\langle m \rangle_{\alpha\beta}$ (for $\alpha, \beta = e, \mu, \tau$) and their corresponding component vectors look like in the complex plane. The relations of such triangles to the Majorana phases and to the lepton-number-violating decays $H^{++} \to \alpha^+ \beta^+$ in the type-II seesaw mechanism are also illustrated.

1. Introduction

In the quark sector the language of the unitarity triangles (UTs) has proved to be quite useful in describing weak CP violation which is governed by the nontrivial phase of the 3×3 Cabibbo–Kobayashi–Maskawa (CKM) quark flavor mixing matrix [1]. The same UT language was first applied to the lepton sector in 1999 [2] to illustrate CP violation in neutrino oscillations, and

* Corresponding author.
E-mail addresses: xingzz@ihep.ac.cn (Z.-z. Xing), zhujingyu@ihep.ac.cn (J.-y. Zhu).

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a peculiar role of the Majorana phases in such leptonic UTs was emphasized in 2000 [3]. Since then a lot of attention has been paid to this kind of geometrical description of lepton flavor mixing and its applications in neutrino phenomenology [4–9].

Thanks to a number of well-established neutrino oscillation experiments [10], one has determined the neutrino oscillation parameters Δm^2_{21}, $|\Delta m^2_{31}|$, θ_{12}, θ_{13} and θ_{23} to a good degree of accuracy in the standard three-flavor scheme [11,12]. Although the sign of Δm^2_{31} remains unknown, a preliminary hint for $\delta \sim 3\pi/2$ has been seen by combining the latest T2K [13] and Daya Bay [14] data [15]. This progress is remarkable, because it allows us to plot the UTs of the 3×3 Pontecorvo–Maki–Nakagawa–Sakata (PMNS) lepton flavor mixing matrix U [16] in the complex plane to show their real shapes. One of the purposes of the present paper is just to do this job. We are going to classify the six UTs into two categories: three Dirac triangles governed by the orthogonality relations

$$\begin{aligned}
\triangle_e: &\quad U_{\mu 1}U^*_{\tau 1} + U_{\mu 2}U^*_{\tau 2} + U_{\mu 3}U^*_{\tau 3} = 0\,, \\
\triangle_\mu: &\quad U_{\tau 1}U^*_{e1} + U_{\tau 2}U^*_{e2} + U_{\tau 3}U^*_{e3} = 0\,, \\
\triangle_\tau: &\quad U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0\,,
\end{aligned} \tag{1}$$

which are insensitive to the Majorana phases of U; and three Majorana triangles dictated by the orthogonality relations

$$\begin{aligned}
\triangle_1: &\quad U_{e2}U^*_{e3} + U_{\mu 2}U^*_{\mu 3} + U_{\tau 2}U^*_{\tau 3} = 0\,, \\
\triangle_2: &\quad U_{e3}U^*_{e1} + U_{\mu 3}U^*_{\mu 1} + U_{\tau 3}U^*_{\tau 1} = 0\,, \\
\triangle_3: &\quad U_{e1}U^*_{e2} + U_{\mu 1}U^*_{\mu 2} + U_{\tau 1}U^*_{\tau 2} = 0\,,
\end{aligned} \tag{2}$$

whose orientations are fixed by the Majorana phases of U. In section 2 the real shapes of these six triangles will be shown with the help of the best-fit results of six neutrino oscillation parameters, and their uncertainties associated with the 1σ uncertainties of the input parameters will be briefly illustrated. Furthermore, the possibilities of right or isosceles UTs in a given neutrino mass ordering will be discussed, and the connections of the Majorana UTs with neutrino–antineutrino oscillations and neutrino decays will be explored.

On the other hand, we are curious about whether the reconstructed elements of the effective Majorana neutrino mass matrix

$$\langle m\rangle_{\alpha\beta} \equiv m_1 U_{\alpha 1}U_{\beta 1} + m_2 U_{\alpha 2}U_{\beta 2} + m_3 U_{\alpha 3}U_{\beta 3} \tag{3}$$

can be similarly described in the complex plane. The answer is affirmative, but this will involve the quadrangles instead of the triangles in general [17]. In the neutrino mass limit $m_1 \to 0$ or $m_3 \to 0$, which is compatible with current neutrino oscillation data and allows one to remove one of the Majorana phases, the relations in Eq. (3) will be simplified to describe six triangles. Such mass triangles (MTs) are phenomenologically interesting in the sense that they are directly related to some rare but important lepton-number-violating (LNV) processes. The example associated with the neutrinoless double-beta ($0\nu 2\beta$) decay has recently been discussed in Ref. [18]. In the present paper we are going to show how each MT formed by $\langle m\rangle_{\alpha\beta}$ (for $\alpha, \beta = e, \mu, \tau$) and its two component vectors in the $m_1 \to 0$ or $m_3 \to 0$ limit looks like. The relations of such triangles to the Majorana phases and to the LNV decays $H^{++} \to \alpha^+\beta^+$ in the type-II seesaw mechanism will also be illustrated.

Let us stress that considering the neutrino mass limit $m_1 \to 0$ or $m_3 \to 0$ makes sense in several aspects. Experimentally, this possibility is not in conflict with any available data. Theoretically, it is consistent with the spirit of Occam's razor [19], and either $m_1 = 0$ or $m_3 = 0$