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Quantization of BMS_3 orbits: A perturbative approach

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Abstract

We compute characters of the BMS group in three dimensions. The approach is the same as that performed by Witten in the case of coadjoint orbits of the Virasoro group in the eighties, within the large central charge approximation. The procedure involves finding a Poisson bracket between classical variables and the corresponding commutator of observables in a Hilbert space, explaining why we call this a quantization. We provide first a pedagogical warm up by applying the method to both $SL(2,\mathbb{R})$ and Poincaré$_3$ groups. As for BMS_3, our results coincide with the characters of induced representations recently studied in the literature. Moreover, we relate the 'coadjoint representations' to the induced representations.

1. Introduction

In the context of Conformal Field Theory in two dimensions and gravity theories in $2+1$ dimensions, the appearance of infinite-dimensional symmetries is inevitable and well understood. The task of investigating the infinite-dimensional groups of such symmetries and their representations is as difficult as important. The main reason for the increasing difficulty is that procedures such as geometric quantization or the construction of induced representations stem from the use of certain invariant (or quasi-invariant) measures on some manifolds (coadjoint orbits), but these

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manifolds are infinite-dimensional and such measures are not known (they may not exist in fact). Other issues also make things harder, as the fact that the standard methods for integrating a Lie algebra representation do not apply [1].

For the particular case of General Relativity in three dimensions one encounters the Virasoro group if a negative cosmological constant is considered, and the so-called BMS_3 group if no cosmological constant is present. These infinite-dimensional Lie groups appear as symmetries of the space of asymptotic solutions and it can be seen that such phase spaces are foliated by the coadjoint orbits of these groups [2,3]. The complete classification of Virasoro orbits was performed in [4] and a more thorough study of the energy function as well as orbit representatives was done in [5]. The same aspects where investigated in [3,6,7] for BMS_3.

A complete quantization of these spaces of solutions, roughly speaking, would start from considering the Poisson manifold that encompasses the union of coadjoint orbits as well as the classical observables on it, and then finding their corresponding quantum operators on a suitable Hilbert space where the classical symmetries are realized as unitary transformations. This is an open problem. What we can do instead is to concentrate on a sector of the classical theory, one coadjoint orbit. For example, in AdS_3 gravity, each BTZ black hole [8] lies in a unique Virasoro coadjoint orbit [2,3] of the type $\mathrm{Diff}(S^1)/S^1$, and one can consider the observables on this symplectic manifold and attempt to give a unitary representation of them on some Hilbert space, thus quantizing this "BTZ sector" of AdS_3 gravity. It also has a mathematical interest on its own, and the complete classification of unitary positive-energy representations of Virasoro group has been recently given by Neeb and Salmasian in [9]. Their work can be regarded, among other things, as a (kind of) geometric quantization of the orbit $\mathrm{Diff}(S^1)/S^1$, since their Hilbert space is given by certain holomorphic sections on a line bundle over the orbit. As far as we know, there is no analogue unitary representations for BMS_3 group.[1]

Despite the difficulties mentioned at the beginning, it is possible to predict meaningful results without a full understanding of the quantum picture. Probably the most relevant one is the spectrum of the Hamiltonian, which can be obtained by computing the character of the time evolution operator, i.e. the partition function. For example, in [4] this character of the Virasoro group is computed by means of a heuristic use of the Lefschetz formula [10]. Remarkably, the same answer is given by a unitary Verma module representation of the Virasoro algebra[2] and also by a perturbative analysis which turns out to give a system of free bosons [4].

As far as we know, there is no general argument explaining why the perturbative quantization (summarized below) gives the same characters as the ones obtained from non-perturbative methods. If it turns out to be the case that it suffices to study the symmetry group in a perturbative fashion in order to compute the characters, then it could also be the case that some other relevant aspects of the theories are fully accessible already at the perturbative level. We are not going to explore this possibility here, but use it as a speculative additional reason to justify the perturbative approach we employ to study the quantization of certain orbits of BMS_3.

The perturbative method for Virasoro orbits of [4] (see also [11] for $\mathrm{Diff}(S^1)/\mathrm{PSL}^{(n)}(2,\mathbb{R})$ orbits), can be used in order to obtain a perturbative quantization of a particular orbit of a different group. We are going to exploit this in order to do such a thing for orbits of BMS_3. This

[1] The BMS_3 induced representations studied in [6] rely on the unproven hypothesis that there exists a quasi-invariant measure on Virasoro orbits.

[2] Although rarely mentioned, it is also a fact that using the Goodman–Wallach unitary representation of Virasoro group [1] over the completion of a unitary Verma module (under its usual inner product) the character is still the same as that coming just from the algebra representation, since the Verma module is trivially dense on its completion.