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Six loop analytical calculation of the field anomalous dimension and the critical exponent η in $O(n)$-symmetric φ^4 model

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Abstract

We report on a completely analytical calculation of the field anomalous dimension γ_φ and the critical exponent η for the $O(n)$-symmetric φ^4 model at the record six loop level. We successfully compare our result for γ_φ with $n = 1$ with the predictions based on the method of the Borel resummation combined with a conformal mapping (Kazakov et al., 1979 [40]). Predictions for seven loop contribution to the field anomalous dimensions are given.

1. Introduction

Since Kenneth Wilson, who was first to apply ϵ-expansion and renormalization group method to calculate critical exponents in φ^4 model, this model became one of the most popular testing grounds for a wide range of methods of diagram calculations and resummation. The first two terms of the ϵ-expansion were calculated by Wilson in [1], ϵ^3 terms and ϵ^4 for critical exponent

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η were calculated in [2]. The latter work was the last where calculations using Wilson renormalization group approach were performed for this model. All subsequent calculations were performed using quantum field renormalization group approach, which effectively reduces the problem of evaluation of critical exponents to the one of finding the corresponding beta-function (or the anomalous dimension).

This approach combined with modern computational techniques allows one to calculate high order corrections with significantly less effort than in the original Wilson's formalism. Using this approach ϵ^4 terms for other exponents were found in [3]. The field anomalous dimension γ_φ and the critical exponent η were calculated with 5-loop accuracy in [4], the 5-loop β-function was first published in [5,6]. Later some (numerically insignificant) inaccuracies were found in this calculation and results for index η and β-function were corrected [7]. Recently, a completely independent check of the analytic results [4–7] was successfully performed in [8] with the use of purely numerical methods.

In this work we describe the results of a completely analytical calculation of γ_φ and η at six loop level in the $O(n)$-symmetric φ^4 model.

2. Setup and notations

The (renormalized) Lagrangian of the φ^4-model in the Euclidean space of $d = 4 - 2\varepsilon$ dimensions reads

$$\mathcal{L}(\varphi) = \frac{1}{2} m^2 Z_1 \varphi^2 + \frac{1}{2} Z_2 (\partial\varphi)^2 + \frac{16\pi^2}{4!} Z_4\, g\, \mu^{2\varepsilon}\, \varphi^4, \tag{1}$$

where RCs (Renormalization Constants) Z_i are expressed in terms of renormalization constants of the field $\varphi_0 = \varphi Z_\varphi$, mass $m_0^2 = m^2 Z_{m^2}$ and coupling constant $g_0 = g\mu^{2\epsilon} Z_g$ in the standard way:

$$Z_1 = Z_{m^2} Z_\varphi^2, \qquad Z_2 = Z_\varphi^2, \qquad Z_4 = Z_g Z_\varphi^4. \tag{2}$$

In the MS-scheme [9] which we employ throughout the paper the UV counterterms do not depend on μ and may depend only *polynomially* on any other dimensionfull parameter of a theory [10]. As a result the RCs Z_i do depend on the regulating parameter ε and renormalized coupling constant g *only* and can be written as:

$$Z_i = 1 + \sum_{k=1} \frac{Z_{i,k}(g)}{\varepsilon^k}. \tag{3}$$

Given the RC $Z_\varphi(g)$, the corresponding anomalous dimension of the scalar field we are interested in is defined as follows

$$\gamma_\varphi(g) = \mu \frac{\partial \log Z_\varphi(g)}{\partial \mu}\Big|_{g_0,\varphi_0} = \beta(g) \frac{\partial \log Z_\varphi}{\partial g} = -2g \frac{\partial Z_{\varphi,1}(g)}{\partial g} = -g\, \frac{\partial Z_{2,1}(g)}{\partial g}. \tag{4}$$

The RC Z_2 and Z_{m^2} are related with UV divergences of the two point one particle irreducible Green function $\Gamma_2(p, m_0^2, g_0)$, which is connected with two point Green function (propagator) $D(p, m_0^2, g_0)$ by Dyson equation $D^{-1}(p, m_0^2, g_0) = p^2 + m_0^2 - \Gamma_2(p, m_0^2, g_0)$. Thus for renormalized two point Green function $D^R(p, m^2, g, \mu)$ we got