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Asymptotic behaviour of two-point functions in multi-species models

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Abstract

We extract the long-distance asymptotic behaviour of two-point correlation functions in massless quantum integrable models containing multi-species excitations. For such a purpose, we extend to these models the method of a large-distance regime re-summation of the form factor expansion of correlation functions. The key feature of our analysis is a technical hypothesis on the large-volume behaviour of the form factors of local operators in such models. We check the validity of this hypothesis on the example of the $SU(3)$-invariant XXX magnet by means of the determinant representations for the form factors of local operators in this model. Our approach confirms the structure of the critical exponents obtained previously for numerous models solvable by the nested Bethe Ansatz.

0. Introduction

Form factor expansions, and hence form factors, play an important role in the characterisation of correlation functions. Over the last few decades, there has been a significant progress in describing the form factors and the associated expansions for so-called quantum integrable systems. First progress in characterising the form factors has been achieved for massive models

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directly in the infinite volume. Archetypes of such models are given by the massive integrable quantum field theories in $1+1$ dimensions. In such a setting, the form factors were characterised as solutions to a set of bootstrap equations [31,33,72]. The resolution of the bootstrap program allowed for an explicit description of the form factors of local operators in numerous models and of some of their intrinsic properties [1,12,26,31,42,51,72]. One should also mention the significant progress in conforming the representation theory of quantum affine algebras to the description of the spectrum of the XXZ spin chain Hamiltonian [13]. This progress allowed to access to multiple integral representations for the form factors in various massive spin chain models [4,30,43], again directly in the infinite volume.

More recently, the calculation of form factors of finite volume quantum integrable models associated to rank one Lie algebras was undertaken within the algebraic Bethe Ansatz [22]. The approach builds on two major ingredients: on the one hand the solution of the quantum inverse scattering problem [39,50] and, on the other hand, determinant representations for the norms [40] and scalar products [67] of Bethe vectors. Within such a setting one obtains finite-size determinant representations for the form factors, see *e.g.* [39,41]. Typically, when the model's volume goes to infinity – the so-called thermodynamic limit –, so does the size of the matrix whose determinant is evaluated. The very structure of the limit depends strongly on whether the underlying model exhibits a massless or a massive spectrum. The massive case is easier to deal with in that, individual form factors decay as integer powers of the volume L. This integer power in L decay allows one to replace discrete sums appearing in a finite volume form factor expansion by a series of multiple integrals, once the thermodynamic limit is taken. The investigation of the large-volume behaviour of a specific form factor in the massive regime of the XXZ chain was carried out in [28]. Later, the analysis was extended to all form factors of the longitudinal spin operator in [19]. The main complication associated with a massless spectrum is that the form factors are expected to vanish as some, generically, non-integer power of the volume [11]. The presence of such vanishing strongly complicates the analysis. First results relative to extracting the leading in L behaviour out of the determinant representations were obtained in [68]. They concerned the form factors of the current operator in the non-linear Schrödinger model. The technique of analysis developed there was improved and extended in [34,36] where the large-volume behaviour of so-called particle–hole form factors in the massless regime of the XXZ chain was obtained. See also [17,18,45] where the analysis of the low-temperature limit of so-called thermal form factors in a massless model at finite temperature has been carried out.

The main issue with the non-integer decay in the volume of individual form factors is that it does not allow one to replace the finite-volume form factor expansions by series of multiple integrals. In fact, for a finite spatial and/or temporal separation between the operators it has been impossible, so far, to write any meaningful form factor series expansion in the thermodynamic limit. Even though intractable in general, form factor series expansion for massless models have recently been discovered to be manageable in the limit of large spatial separations between the operators building up the correlator from which the expansion originates. Indeed, when the volume is large but finite, the large-distance/time asymptotic behaviour of such series can be extracted by means of a variant of the saddle-point method. The evaluation of the leading contribution to the correlator is achieved through the evaluation of certain multidimensional sums over the massless excitations of the model. After re-summing, one can already take the infinite volume limit, hence accessing to the large-distance asymptotic behaviour of the correlator. This approach has been developed in [35,37,38] and culminated in the construction of a direct mapping [46] between the zero energy excitation sector of a massless model and the free boson field theory.