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Spectra of sigma models on semi-symmetric spaces

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Abstract

Sigma models on semi-symmetric spaces provide the central building block for string theories on *AdS* backgrounds. Under certain conditions on the global supersymmetry group they can be made one-loop conformal by adding an appropriate fermionic Wess–Zumino term. We determine the full one-loop dilation operator of the theory. It involves an interesting new XXZ-like interaction term. Eigenvalues of our dilation operator, i.e. the one-loop anomalous dimensions, are computed for a few examples.

1. Introduction

Non-linear sigma models (NLSM), such as the famous $O(N)$ or $\mathbb{CP}^N$ models, play an important role in high and low energy physics as well as mathematics. For a long time, research focused on cases in which the target is a symmetric space, i.e. can be written as a quotient G/H of a (super)group G by a subgroup $H \subset G$ that is invariant under the action of an involution $\sigma : G \to G$, i.e. by an automorphism of order two. The AdS/CFT correspondence has brought more general homogeneous spaces into the spotlight, including

$$PSU(2,2|4)\,/\,SO(1,4) \times SO(5) \quad \text{and} \quad SPO(2,2|6)\,/\,SO(1,3) \times U(3)$$

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which describe strings moving on $AdS_5 \times S^5$ and $AdS_4 \times \mathbb{CP}^3$, respectively. These spaces are examples of so-called generalized symmetric spaces G/H. By definition, the stabilizer sub(super)group H of a generalized symmetric space is left invariant by the action of an automorphism σ of order $M > 2$. Sigma models on such generalized symmetric spaces are not uniquely fixed by the target space manifold since the G symmetry leaves us with some $M - 1$ dimensional space of metrics. Additionally, it is possible to add θ or Wess–Zumino (WZ) terms.

Since all the known relevant examples, such as the two displayed above, involve automorphisms or order four, we shall restrict to $M = 4$. The corresponding coset spaces G/H are often referred to as semi-symmetric and their sigma models appear as a part of the world-sheet action for strings in homogeneous AdS backgrounds, regardless of whether one works within the Green–Schwarz [1–5], pure spinor [6] or hybrid formalism [7,8]. Only the choice of couplings depends on which formulation of superstring theory is actually being used. In this paper we are not concerned with the relations between the different approaches [9–14] and simply pick the couplings in the action such that we recover the NLSM of the hybrid and pure spinor models. In these cases, the metric conspires with a fermionic WZ term in order to make the action classically integrable and the one-loop beta-function vanish [15].

The aim of this work is to study the spectrum of one-loop anomalous dimensions for one-loop conformal NLSMs on semi-symmetric superspaces. Our analysis is valid for all such models, compact and non-compact. In order to fully control the one-loop spectrum, one also needs to enumerate possible vertex operators in the free theory. This is a problem of harmonic analysis which will not be addressed in the present work, see [16], however, where the corresponding issue has been solved for compact supercoset geometries. The analysis of [16] carries over to non-compact cosets as long as the stabilizer subgroup H is compact and a generalization to noncompact H is possible at least on a case-by-case basis.

Our results for the full one-loop dilation operator of one-loop conformal NLSMs on semi-symmetric spaces will be given in section 4. For non-derivative fields (zero modes) the one-loop dilation operator is given by the Bochner Laplacian, just as in the case of symmetric superspaces, see [16]. For operators including world-sheet derivatives the results becomes more interesting. In this case, the one-loop dilation operator turns out to involve an interesting new XXZ-like interaction term that we introduce in section 4 and derive in section 5. The interacting spins take values in space of tangent vectors to the semi-symmetric background. We shall also evaluate the general formulas for fields involving two derivatives. In this case the dilation operator can be diagonalized easily so that we can read off the anomalous dimensions.

The structure of the paper is as follows. In section 2 we will recall some facts of sigma models on semi-symmetric spaces, focusing on the one-loop conformal case that appears as part of the action in hybrid and pure spinor models. We will also present the one-loop expansion of the action. In section 3 we briefly describe how to build fields for the model and we spell out the leading terms of these vertex operators in the background field expansion. Section 4 contains the main result of this work. There we describe the full one-loop dilation operator and we analyze the anomalous dimensions for a particular subset of fields. The results are then derived in the section 5. Auxiliary integral formulas, finally, are collected in an appendix at the end of the paper.

2. Sigma models on semi-symmetric spaces

The goal of this section is to introduce and discuss the models we are dealing with. In the first subsection we construct their action. Their background field expansion is the subject of the