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Exact cosmological solutions of $f(R)$ theories via Hojman symmetry

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Abstract

Nowadays, $f(R)$ theory has been one of the leading modified gravity theories to explain the current accelerated expansion of the universe, without invoking dark energy. It is of interest to find the exact cosmological solutions of $f(R)$ theories. Besides other methods, symmetry has been proved as a powerful tool to find exact solutions. On the other hand, symmetry might hint the deep physical structure of a theory, and hence considering symmetry is also well motivated. As is well known, Noether symmetry has been extensively used in physics. Recently, the so-called Hojman symmetry was also considered in the literature. Hojman symmetry directly deals with the equations of motion, rather than Lagrangian or Hamiltonian, unlike Noether symmetry. In this work, we consider Hojman symmetry in $f(R)$ theories in both the metric and Palatini formalisms, and find the corresponding exact cosmological solutions of $f(R)$ theories via Hojman symmetry. There exist some new solutions significantly different from the ones obtained by using Noether symmetry in $f(R)$ theories. To our knowledge, they also have not been found previously in the literature. This work confirms that Hojman symmetry can bring new features to cosmology and gravity theories.

1. Introduction

The current accelerated expansion of the universe could be due to an unknown energy component (dark energy) or a modification to general relativity (modified gravity) [1,2]. In the

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literature, various modified gravity theories were proposed to account for the cosmic acceleration, such as $f(R)$ theory [2–4,45,48], scalar-tensor theory [4,5], braneworld model [6,7], Galileon gravity [8,9], Gauss–Bonnet gravity [10], $f(T)$ theory [11,12], massive gravity [13,14]. Nowadays, modified gravity theories have been one of the main fields in modern cosmology.

As one of the leading modified gravity theories, $f(R)$ theory was proposed by generalizing the well-known Einstein–Hilbert Lagrangian R used in general relativity (GR) to an arbitrary function $f(R)$, where R is the scalar curvature. In fact, $f(R)$ theory has been extensively studied in the literature for many years (see e.g. [2–4] for reviews). It can be used to drive inflation (see e.g. [15]), play the role of dark matter (see e.g. [16]), or drive the current accelerated expansion of the universe as an competitive alternative of dark energy (see e.g. [17,18]).

Note that there exist two different types of $f(R)$ theories in the literature (see e.g. [2–4]), namely $f(R)$ theory in the metric formalism, and $f(R)$ theory in the Palatini formalism. In the metric formalism, the affine connection $\Gamma^{\lambda}_{\alpha\beta}$ depends on the metric $g_{\mu\nu}$, and hence the field equations are derived by the variation of the action with respect to the metric $g_{\mu\nu}$ only. On the other hand, in the Palatini formalism, the affine connection $\Gamma^{\lambda}_{\alpha\beta}$ and the metric $g_{\mu\nu}$ are treated as independent variables when one varies the action. As is well known, in the case of GR (namely $f(R) \propto R$), the field equations are completely identical in these two formalisms. However, in the case of non-linear $f(R)$, the field equations are different in these two formalisms. So, the metric and Palatini $f(R)$ theories should be considered separately.

It is of interest to find the exact cosmological solutions of $f(R)$ theories. Besides other methods (e.g. reconstruction [47]), symmetry has been proved as a powerful tool to find exact solutions. On the other hand, symmetry might hint the deep physical structure of a theory, and hence considering symmetry is also well motivated. As is well known, Noether symmetry has been extensively used in cosmology and gravity theories, for instance, scalar field cosmology [19,20], $f(R)$ theory [21–25,46,48], scalar-tensor theory [26,27], $f(T)$ theory [28,29], Gauss–Bonnet gravity [30], non-minimally coupled cosmology [31], and others [32]. It is worth noting that a (point-like) Lagrangian should be given *a priori* when one uses Noether symmetry.

In this work, we are interested to consider the so-called Hojman symmetry in $f(R)$ theories, and find the corresponding exact cosmological solutions of $f(R)$ theories via Hojman symmetry. Unlike Noether conservation theorem, the symmetry vectors and the corresponding conserved quantities in Hojman conservation theorem can be obtained by using the equations of motion directly, without using Lagrangian or Hamiltonian. In general, its conserved quantities and the exact solutions can be quite different from the ones via Noether symmetry. In fact, recently Hojman symmetry has been used in cosmology and gravity theory [33–35]. It is found that Hojman symmetry exists for a wide range of the potential $V(\phi)$ of quintessence [33] and scalar-tensor theory [34], and the corresponding exact cosmological solutions have been obtained. While Noether symmetry exists only for exponential potential $V(\phi)$ [19,26,27], Hojman symmetry can exist for a wide range of potentials $V(\phi)$, including not only exponential but also power-law, hyperbolic, logarithmic and other complicated potentials [33,34]. On the other hand, it is also found that Hojman symmetry exists in $f(T)$ theory and the corresponding exact cosmological solutions are obtained [35]. The functional form of $f(T)$ is restricted to be the power-law or hypergeometric type, while the universe experiences a power-law or hyperbolic expansion. These results are also different from the ones obtained by using Noether symmetry in $f(T)$ theory [28]. Therefore, although some exact cosmological solutions of $f(R)$ theories were found by using Noether symmetry in the literature [21–25], it is still interesting to find them by using Hojman symmetry instead, because as mentioned above one can expect that the solutions via Hojman symmetry