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Calculating TMDs of a large nucleus: Quasi-classical approximation and quantum evolution

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Abstract

We set up a formalism for calculating transverse-momentum-dependent parton distribution functions (TMDs) of a large nucleus using the tools of saturation physics. By generalizing the quasi-classical Glauber–Gribov–Mueller/McLerran–Venugopalan approximation to allow for the possibility of spin–orbit coupling, we show how any TMD can be calculated in the saturation framework. This can also be applied to the TMDs of a proton by modeling it as a large "nucleus." To illustrate our technique, we calculate the quark TMDs of an unpolarized nucleus at large-x: the unpolarized quark distribution and the quark Boer–Mulders distribution. We observe that spin–orbit coupling leads to mixing between different TMDs of the nucleus and of the nucleons. We then consider the evolution of TMDs: at large-x, in the double-logarithmic approximation, we obtain the Sudakov form factor. At small-x the evolution of unpolarized-target quark TMDs is governed by BK/JIMWLK evolution, while the small-x evolution of polarized-target quark TMDs appears to be dominated by the QCD Reggeon.

1. Introduction

Over the past decade quark and gluon transverse momentum-dependent parton distribution functions (TMDs) [1,2] have become an integral component of our understanding of the

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momentum-space structure of the nucleon. At the same time the main principles for calculating TMDs in the perturbative QCD framework have remained essentially the same over the years: one parameterizes the initial conditions at some initial virtuality $Q^2 = Q_0^2$ and then applies the Collins–Soper–Sterman (CSS) evolution equation [1] to find the TMDs at all Q^2. The initial conditions are non-perturbative, and have to be constructed using models of the non-perturbative QCD dynamics (see [3] and references therein). Since very little is known about non-perturbative effects in QCD, often one uses the same form for the parameterization of the initial conditions for several different TMDs, frequently assuming a Gaussian dependence of the TMDs on the parton transverse momentum k_T [4]. It is clearly desirable to have a better control of our qualitative and quantitative understanding of TMDs.

To this end one can employ the recent progress in our understanding of small-x physics and parton saturation [5–11] to put constraints on the TMDs and even calculate them in the high-energy limit. When calculating TMDs one often works either in the $s \sim Q^2 \gg k_T^2$ (large-x) or in the $s \gg Q^2 \gg k_T^2$ (small-x) regimes, where s is the center-of-mass energy and $x = Q^2/(s + Q^2)$ if one neglects the proton mass. In either case the energy s is large, and the techniques of high-energy QCD should apply. The degrees of freedom in saturation physics are infinite Wilson lines along (almost) light-like paths. The definition of the TMDs involves light-cone Wilson lines as well [12,13], although the integration paths are semi-infinite, forming the so-called "light-cone staple". We can see that there are both similarities and differences between saturation physics and the physics of TMDs. The interface of these two sub-fields of quantum chromodynamics (QCD) has been explored in [14–24].

In the past some success has been achieved in applying saturation physics to study the Sivers function [25,26] both in semi-inclusive deep inelastic scattering (SIDIS) and in the Drell–Yan process (DY). In [27] the Sivers function was constructed by generalizing the quasi-classical Glauber–Gribov–Mueller (GGM) [28]/McLerran–Venugopalan (MV) [29–31] approximation of a heavy nucleus with atomic number $A \gg 1$. The presence of the atomic number generates a resummation parameter $\alpha_s^2 A^{1/3}$ [32,33] allowing a systematic resummation of multiple rescatterings, which are essential for the Sivers function. This picture can also be applied to the proton if one models it as a large "nucleus." This large-nucleus approximation is known to work well in describing the data from deep inelastic scattering (DIS) experiments on a proton at low-x; it is therefore possible that it would give a reasonable description for proton TMDs as well. The result of [27] was an explicit form of the Sivers function in the $s \sim Q^2 \gg k_T^2$ regime, which was different from a simple Gaussian in k_T and which can be used as the initial condition for its Collins–Soper–Sterman (CSS) evolution [1]. Another important result of [27] was the realization that the Sivers function can be produced via two different channels: one is the standard "lensing" mechanism of [34–36] with additional momentum broadening due to multiple rescatterings in the nucleus, while the other mechanism was due to the orbital angular momentum (OAM) of the nucleus combined with multiple rescatterings. This latter channel has not been reported before [27], and it dominates mainly in the regime where multiple rescatterings are important, or, more precisely, for $k_T \lesssim Q_s/\sqrt{\alpha_s}$, where Q_s is the saturation scale of the nucleus. It appears that applications of saturation physics to the calculation of TMDs may lead to qualitatively new channels of generating the relevant observables.

The aim of this work is twofold. First of all we want to generalize the approach of [27] to the calculation of any TMD in the quasi-classical approximation and for $s \sim Q^2 \gg k_T^2$. This is accomplished in Sec. 2 for the case of an unpolarized nucleus; the generalization to the polarized case is straightforward and is left for future work. The quasi-classical TMD calculation is accomplished using the factorization given in Eq. (18) (and, in more detail in Eq. (39)), which