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Scale invariance of the η-deformed $AdS_5 \times S^5$ superstring, T-duality and modified type II equations

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Abstract

We consider the ABF background underlying the η-deformed $AdS_5 \times S^5$ sigma model. This background fails to satisfy the standard IIB supergravity equations which indicates that the corresponding sigma model is not Weyl invariant, i.e. does not define a critical string theory in the usual sense. We argue that the ABF background should still define a UV finite theory on a flat 2d world-sheet implying that the η-deformed model is scale invariant. This property follows from the formal relation via T-duality between the η-deformed model and the one defined by an exact type IIB supergravity solution that has 6 isometries albeit broken by a linear dilaton. We find that the ABF background satisfies candidate type IIB scale invariance conditions which for the R–R field strengths are of the second order in derivatives. Surprisingly, we also find that the ABF background obeys an interesting modification of the standard IIB supergravity equations that are first order in derivatives of R–R fields. These modified equations explicitly depend on Killing vectors of the ABF background and, although not universal, they imply the universal scale invariance conditions. Moreover, we show that it is precisely the non-isometric dilaton of the T-dual solution that

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leads, after T-duality, to modification of type II equations from their standard form. We conjecture that the modified equations should follow from κ-symmetry of the η-deformed model. All our observations apply also to η-deformations of $AdS_3 \times S^3 \times T^4$and $AdS_2 \times S^2 \times T^6$models.

1. Introduction

The study of integrable deformations of the $AdS_5 \times S^5$ superstring sigma model is an important direction in the search for new solvable examples of AdS/CFT duality. An interesting one-parameter integrable generalisation of the classical $AdS_5 \times S^5$ Green–Schwarz action related to the quantum deformation of the underlying supergroup symmetry was found in [1]. Just from the construction of this "η-model" (based on a particular current–current deformation of the supercoset action [2] generalising the bosonic model of [3]) there is no a priori reason why it should define a scale invariant (UV finite) 2d theory and, moreover, why it should preserve the conformal (Weyl) invariance and hence still correspond to a consistent superstring theory as the undeformed $AdS_5 \times S^5$ model does.[3]

The only indication in this direction is that the η-model action, like the original $AdS_5 \times S^5$ action, is invariant under a version of fermionic κ-symmetry [1], which reduces the number of fermions by half. However, the usual claim that κ-symmetry implies the corresponding action can be interpreted as that of a GS superstring propagating in a background that is a consistent type II supergravity solution (and hence defines a consistent critical superstring theory) assumes that the κ-symmetry is of the standard GS "projector" form [5]. This is most probably not the case for the η-model at higher orders in fermions. Indeed, it was found in [6,7] that the target space background corresponding to the η-model action [1], interpreted as a GS action, does not represent a type IIB supergravity solution.

Starting with the GS Lagrangian written in superspace form ($Z^M = (x^m, \theta^\alpha)$)

$$L = (\sqrt{h} h^{ab} E^r_M E^s_N \eta_{rs} - \epsilon^{ab} B_{MN}) \partial_a Z^M \partial_b Z^N \ , \qquad (1.1)$$

one can solve the standard type II superspace constraints and Bianchi identities for $E(Z)$, $B(Z)$ (which imply the supergravity equations) in order to express the GS action in terms of component fields. One then observes that the dilaton ϕ (which is part of the dilaton superfield $\Phi(Z)$ that is introduced in the process of solving the constraints) enters the world-sheet action (i) in the combination $\mathcal{F} = e^\phi F$ with the R–R field strengths starting at order θ^2 and (ii) via derivatives $\partial_m \phi$ starting at order θ^4 (see [8] and the references therein). This action has classical Weyl invariance and κ-symmetry, which will be broken, in general, by quantum corrections. As for the bosonic string [9], to cancel the 2d stress tensor trace anomaly requires adding the familiar 1-loop dilaton counterterm $\sim \int d^2 z \sqrt{h} R^{(2)} \Phi(Z)$ (see [10,11] and the references therein).[4]

The case relevant to our discussion below is a special isometric type II solution for which the metric G_{mn}, B-field B_{mn} and R–R fields $\mathcal{F}_{m_1 \ldots m_n}$ are invariant while ϕ is linear in the isometric

[3] This is in contrast, e.g., to the integrable deformation [4] based on TsT duality transformations, which preserve conformality. In particular, the TsT deformed background is a solution of type IIB supergravity.

[4] This additional term is certainly required to reproduce the standard 1-loop Weyl-invariance conditions for the G and B-field couplings or supergravity equations in NS–NS sector. This term should also be required to cancel the quantum anomaly of κ-symmetry.