Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 903 (2016) 400–417

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Ultraspinning limits and rotating hyperboloid membranes

Robie A. Hennigar [a,*], David Kubizňák [b,a], Robert B. Mann [a], Nathan Musoke [b]

[a] *Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada*
[b] *Perimeter Institute, 31 Caroline St. N., Waterloo, Ontario, N2L 2Y5, Canada*

Received 15 December 2015; accepted 30 December 2015

Available online 6 January 2016

Editor: Stephan Stieberger

Abstract

We apply the hyperboloid membrane limit to the general Kerr-AdS metrics and their recently studied super-entropic cousins and obtain a new class of rotating black holes, for which the rotational parameters in multiple directions attain their maximal value—equal to the AdS radius. These new solutions have a potential application in the description of holographic fluids with vorticity. They also possess interesting thermodynamic properties: we show that—despite the absence of Misner strings—the Bekenstein–Hawking entropy/area law is still violated, raising a question about the origin of this violation.

1. Introduction

The study of general relativity in higher dimensions has remained an active and popular field, in part motivated by its applicability in string theory and various gauge/gravity dualities, and also due to the deeper perspective it provides for our understanding of four dimensional gravity [1]. Of particular interest are higher dimensional black hole solutions, whose physics highlights the

* Corresponding author.
E-mail addresses: rhennigar@uwaterloo.ca (R.A. Hennigar), dkubiznak@perimeterinstitute.ca (D. Kubizňák), rbmann@uwaterloo.ca (R.B. Mann), nmusoke@perimeterinstitute.ca (N. Musoke).

http://dx.doi.org/10.1016/j.nuclphysb.2015.12.017

peculiarities of four dimensions. For example, unlike their four-dimensional cousins described by the Kerr solution, the higher dimensional asymptotically flat, rotating black holes of Myers and Perry solve Einstein's equations with arbitrarily large angular momentum for a given mass [2]. When possessing large angular momentum, these black holes are said to be *ultraspinning*.

Ultraspinning limits of rotating black holes have been the attention of numerous studies. Emparan and Myers [3] found that in $d \geq 6$, ultraspinning Myers–Perry black holes present an instability reminiscent of the Gregory–Laflamme instability for black branes [4]. Arguably more interesting is the case of Kerr black holes in asymptotically anti de Sitter (AdS) space, where the 'ultraspinning limit' corresponds to the case where the rotation parameter, a, approaches the AdS radius, l.[1] Here a variety of ultraspinning solutions can be obtained depending on how the $a \to l$ limit is performed. Caldarelli et al. [6] studied the *black brane limit* wherein the physical mass is held fixed and the $a \to l$ limit is taken while simultaneously zooming in to the pole. This limit is sensible only in $d \geq 6$ and yields a static, asymptotically flat black brane. Armas and Obers later demonstrated that the same limiting solution is obtained by taking $a \to \infty$ while keeping the ratio a/l fixed, their approach having the advantage of being directly applicable to dS solutions as well [5]. Caldarelli et al. have also studied the *hyperboloid membrane limit* where the horizon radius, r_+, is held fixed while zooming in to the pole and taking $a \to l$ [6,7]. This limit, sensible for $d \geq 4$, yields a rotating AdS hyperboloid membrane with horizon topology $\mathbb{H}^2 \times S^{d-4}$. Klemm et al. and the present authors have studied the *super-entropic limit* in which one works in rotating coordinates, rescales an azimuthal coordinate to absorb any singular quantities from the $a \to l$ limit, and then compactifies the resulting azimuthal coordinate [8–11]. This limit, sensible for $d \geq 4$, is so-named because the resulting solutions are super-entropic for some values of the parameters, that is, their entropy exceeds, for a given thermodynamic volume, the bound set by the *reverse isoperimetric inequality* [12].

Recently, ultraspinning AdS black holes have found application via the *fluid/gravity correspondence*. This powerful duality relates the gravitational degrees of freedom residing on the boundary of a d-dimensional asymptotically AdS spacetime to the hydrodynamics of a relativistic fluid in $d - 1$ spacetime dimensions (see, e.g., review [13]). On the hydrodynamics side, this paradigm has been used to understand the role of quantum anomalies in hydrodynamical transport [14] as well as understanding the fluid dynamics of superfluids [15]. On the gravity side it has been utilized in the construction of static, inhomogeneous black hole solutions [16] and as a means to classify the possible black hole solutions of Einstein–Maxwell-AdS theory [17].

As progress continues to be made, there are indications that this formalism may prove useful as a means of understanding rotating Bose/Fermi gases via holography [18] as well as turbulence and wave propagation in metamaterials [19]. Important advances in this direction have been spearheaded by Leigh et al. [20] and elaborated upon in later research [7,21,22], where a holographic treatment of fluids with vorticity was given. Within this context, it was found that the conformal boundary of the spacetime obtained via the hyperboloid membrane limit of the 4d Kerr-AdS solution is of Papapertrou–Randers form and can be cast in the Zermelo frame [7]. Such metrics are referred to as acoustic/optical since they describe the propagation of sound/light in relativistic fluids [23], and feature prominently in descriptions of analogue gravity [24].

In this paper we apply the hyperboloid membrane limit to the general Kerr-AdS metrics in all dimensions, thereby obtaining new ultraspinning black hole solutions. We find that for a black

[1] In what follows we shall take this as a definition of the ultraspinning limit of rotating AdS black objects. Note, however, that this is different from the definition used in [5] and the resulting solutions do not necessarily possess (for a given mass) large angular momentum.