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On the soft limit of closed string amplitudes with massive states

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Abstract

We extend our analysis of the soft behavior of string amplitudes with massive insertions to closed strings at tree level (sphere). Relying on our previous results for open strings on the disk and on KLT formulae we check universality of the soft behavior for gravitons to sub-leading order for superstring amplitudes and show how this gets modified for bosonic strings. At sub-sub-leading order we argue in favor of universality for superstrings on the basis of OPE of the vertex operators and gauge invariance for the soft graviton. The results are illustrated by explicit examples of 4-point amplitudes with one massive insertion in any dimension, including $D = 4$, where use of the helicity spinor formalism drastically simplifies the expressions. As a by-product of our analysis we confirm that the 'single valued projection' holds for massive amplitudes, too. We briefly comment on the soft behavior of the anti-symmetric tensor and on loop corrections.

1. Introduction and motivations

The connection between 'gravitational memory', 'soft behavior' of graviton scattering amplitudes and 'BvBMS symmetry' [1–6] seems to play a crucial in a recently proposed solution to the Information Paradox for Black Holes [7]. While waiting for a refined version of the argument, it is natural to ask the fate of the universal 'soft' behavior of graviton scattering amplitudes

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in a quantum theory of gravity such as closed string theory. The problem has been addressed for tree-level amplitudes with only mass-less gravitons in [8,9], relying on KLT formulae and OPE of the vertex operators, and in [10], relying on gauge invariance. Bosonic amplitudes with tachyons have been investigated to sub-leading order in [11,12].

In gravity theories, when one of the external graviton momenta goes soft *i.e.* $k \to 0$ with $k = \delta\hat{k}$ with $\hat{k}$ some fixed momentum, not only the leading δ^{-1} and sub-leading behaviors δ^0 [13,14], but also the next-to-subleading or sub-sub-leading behavior δ^{+1} is universal [15]. Calling $h_s^{\mu\nu}$ the soft graviton polarization and k_s^μ its soft momentum, one has

$$\mathcal{M}_n(1,2,\ldots,s,\ldots,n) \approx \sum_{i\neq s}\left[\frac{k_i\cdot h_s\cdot k_i}{k_s\cdot k_i} + \frac{k_i\cdot h_s\cdot J_i\cdot k_s}{k_s\cdot k_i} + \frac{k_s\cdot J_i\cdot h_s\cdot J_i\cdot k_s}{2k_s\cdot k_i}\right]\mathcal{M}_{n-1}(1,2,\ldots\hat{s}\ldots,n) + \mathcal{O}(\delta^2) \qquad (1)$$

where k_i and J_i denote the 'hard' momenta and angular momentum operators. These results are valid at tree-level and are derived with the understanding that interactions be governed by minimal coupling.

In theories with closed strings, the conclusions, though quite independent of the number of (non-compact) space-time dimensions, depend on the nature of the higher derivative couplings [8]. R^3 terms do not change the universal soft behavior of minimal coupling, while ϕR^2 do modify even the leading term when ϕ is a massless scalar such as the dilaton. This happens in particular in the bosonic string and heterotic string at tree level[1] and in the Type II compactifications preserving less than maximal super-symmetry.

The aim of the present investigation, that may be considered a follow up of [16], is to show that inclusion of massive external states does not spoil the universal 'soft' behavior (1) for Type II theories with maximal susy at tree level. In [16] open string amplitudes with massive external states as well as tachyons have been computed and shown to expose the expected behavior even when non-minimal interactions are considered. Neither F^3 terms nor the coupling $\alpha'\mathcal{T}F^2$, where $\mathcal{T}$ is the tachyon, change the universal soft behavior, based on minimal coupling. On the other hand ϕF^2 terms do modify even the leading term when ϕ is a massless scalar. For color-ordered string amplitudes one gets the same universal behavior as in YM theories [17–29]

$$\mathcal{A}_n(1,2,\ldots,s,\ldots,n) \approx \left\{\left[\frac{a_s\cdot k_{s+1}}{k_s\cdot k_{s+1}} - \frac{a_s\cdot k_{s-1}}{k_s\cdot k_{s-1}}\right] + \left[\frac{f_s:J_{s+1}}{k_s\cdot k_{s+1}} - \frac{f_s:J_{s-1}}{k_s\cdot k_{s-1}}\right]\right\}\mathcal{A}_{n-1}(1,2,\ldots\hat{s}\ldots,n) + \mathcal{O}(\delta) \qquad (2)$$

where a_s and k_s denote the soft gluon polarization and momentum, so that $f_s^{\mu\nu} = k_s^\mu a_s^\nu - k_s^\mu a_s^\nu$ is its linearized field strength, while $k_{s\pm1}$ and $J_{s\pm1}$ denote the 'hard' momenta and angular momentum operators of the adjacent insertions. Relying on [16] and on KLT formulae, we presently analyze closed string amplitudes with massive external states. In the bosonic string case we will also consider tachyons as external states.

Amplitudes with massive external states have been considered earlier on [30–33], see also [34] for the case of 'light' string states and [35–41] as well as the review [42] for more phenomenological applications. The plan of the paper is as follows.

[1] M. B. would like to thank I. Antoniadis for stressing the tree level origin of this term in the heterotic string, which only gets generated at one-loop in 4-dim Type II theories with 16 supercharges, such as after compactification on $K3 \times T^2$. R^3 term is forbidden due to supersymmetry.