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Coset construction of a D-brane gauge field

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Abstract

D-branes have a world-volume $U(1)$ gauge field A whose field strength $F = dA$ gives rise to a Born–Infeld term in the D-brane action. Supersymmetry and kappa symmetry transformations of A are traditionally inferred by the requirement that the Born–Infeld term is consistent with both supersymmetry and kappa symmetry of the D-brane action. In this paper, we show that integrability of the assigned supersymmetry transformations leads to an extension of the standard supersymmetry algebra that includes a fermionic central charge. We construct a superspace one-form on an enlarged superspace related by a coset construction to this centrally extended algebra whose supersymmetry and kappa symmetry transformations are derived, rather than inferred. It is shown that under pullback, these transformations are of the form expected for the D-brane $U(1)$ gauge field. We relate these results to manifestly supersymmetric approaches to construction of D-brane actions.

1. Introduction

In the Green–Schwarz formulation [1–4], p-branes are embeddings of a $(p+1)$-dimensional bosonic world-volume into a superspace,

$$\sigma^i \to \left(x^a(\sigma), \theta^\alpha(\sigma)\right), \tag{1.1}$$

where σ^i are coordinates on the world-volume, and (x^a, θ^α) are superspace coordinates. Here we consider flat D-dimensional $\mathcal{N} = 1$ superspace. The supersymmetry algebra

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$$\{Q_\alpha, Q_\beta\} = -2\,(C\Gamma^a)_{\alpha\beta}\,P_a \tag{1.2}$$

is realised via the transformations of superspace coordinates[1]

$$\delta_\epsilon\, x^a = i\,(\bar\epsilon\Gamma^a\theta) \tag{1.3}$$
$$\delta_\epsilon\,\theta^\alpha = \epsilon^\alpha. \tag{1.4}$$

The one-forms

$$\pi^a = dx^a - i(\bar\theta\Gamma^a d\theta), \quad d\theta^\alpha \tag{1.5}$$

are invariant under supersymmetry transformations. p-brane actions exist only in spacetime dimensions for which the supersymmetry invariant $(p+2)$ form

$$h^{(p+2)} = \pi^{a_1}\wedge\cdots\wedge\pi^{a_p}\,(d\bar\theta\,\Gamma_{a_1}\cdots\Gamma_{a_p}\,d\theta) \tag{1.6}$$

is closed, requiring the gamma matrix identities

$$0 = (C\Gamma^a)_{\alpha(\beta}\,(C\Gamma_a)_{\gamma\delta)}, \quad p=1; \tag{1.7}$$
$$0 = (C\Gamma^{a_1})_{(\alpha\beta}\,(C\Gamma_{a_1\cdots a_p})_{\gamma\delta)}, \quad p>1. \tag{1.8}$$

Here, $\Gamma_{a_1\cdots a_p}$ is the anti-symmetrized product of gamma matrices, and the round brackets on spinor indices denote symmetrisation. The resulting restrictions on p and D give rise to the "brane-scan" [5]. Closure of $h^{(p+2)}$ implies

$$h^{(p+2)} = d\,b^{(p+1)}. \tag{1.9}$$

The p-brane action is

$$S = S_0 + S_{WZ}, \tag{1.10}$$

where the "kinetic" term

$$S_0 = \int d^{(p+1)}\sigma\,\sqrt{det\, G_{ij}} \tag{1.11}$$

is constructed from the pulled back world-volume metric

$$G_{ij} = \pi_i{}^a\eta_{ab}\pi_j{}^b, \tag{1.12}$$

with $\pi_i{}^a = \frac{\partial x^a}{\partial\sigma^i} - i(\bar\theta\Gamma^a\frac{\partial\theta}{\partial\sigma^i})$. The Wess–Zumino term is given by the integral over the $(p+1)$-dimensional world-volume of the pullback of the superspace form $b^{(p+1)}$,

$$S_{WZ} = \int \sigma^* b^{(p+1)}. \tag{1.13}$$

The fact that $b^{(p+1)}$ is not invariant under supersymmetry transformations, but varies by a total derivative, leads to a central extension to the supersymmetry algebra of the form [6]

$$\{Q_\alpha, Q_\beta\} = -2\,(C\Gamma^a)_{\alpha\beta}\,P_a + (C\Gamma_{a_1\cdots a_p})_{\alpha\beta}\,Z^{a_1\cdots a_p}, \tag{1.14}$$

where $Z^{a_1\cdots a_p}$ are bosonic central charges.

[1] Unless otherwise stated, spinors are Majorana, and $\bar\epsilon = \epsilon^T C$, where C is the charge conjugation matrix. Also, the spacetime dimension must be such that $(C\Gamma^a)_{\alpha\beta}$ is symmetric.