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Heisenberg symmetry and hypermultiplet manifolds

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Abstract

We study the emergence of Heisenberg (Bianchi II) algebra in hyper-Kähler and quaternionic spaces. This is motivated by the rôle these spaces with this symmetry play in $\mathcal{N} = 2$ hypermultiplet scalar manifolds. We show how to construct related pairs of hyper-Kähler and quaternionic spaces under general symmetry assumptions, the former being a zooming-in limit of the latter at vanishing scalar curvature. We further apply this method for the two hyper-Kähler spaces with Heisenberg algebra, which is reduced to $U(1) \times U(1)$ at the quaternionic level. We also show that no quaternionic spaces exist with a strict Heisenberg symmetry – as opposed to Heisenberg $\ltimes U(1)$. We finally discuss the realization of the latter by gauging appropriate $Sp(2, 4)$ generators in $\mathcal{N} = 2$ conformal supergravity.

Introduction

In string theory, the Heisenberg algebra appears within the universal hypermultiplet of type IIA compactification [1]. The dilaton is contained in the scalar manifold, which is a four-

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dimensional quaternionic space [2].[1] At tree level, the latter is $\widetilde{\mathbb{C}P}_2$ equipped with the Kähler (non-compact) Fubini–Study metric of $SU(1,2)/U(2)$. Perturbative corrections break the large isometry group of this space to the Heisenberg group, generated by three shifts (NSNS axion and RR scalar) [3]. More precisely, it was observed that the residual symmetry is rather Heisenberg $\ltimes U(1)$, and that this symmetry uniquely determines the quaternionic space. Non-perturbative corrections further break the Heisenberg symmetry down to $U(1) \times U(1)$ [4] (or generically to a discrete subgroup of the Heisenberg group – see for example [5]). The corresponding scalar manifold is thus a quaternionic space with two commuting Killing vectors. Metrics on these manifolds have been characterized by Calderbank and Pedersen [6].

In the above framework, supersymmetry is locally realized and the scalar curvature of the quaternionic space is directly proportional to the gravitational constant $k^2 = 8\pi M_{\text{Planck}}^{-2}$ [2]. For hypermultiplets of global $\mathcal{N} = 2$, the relevant sigma-model target spaces are hyper-Kähler [7]. These Kähler spaces are Ricci-flat and, in the four-dimensional case, Riemann self-dual i.e. they are gravitational instantons. There exists then plausibly a low-energy decoupling limit of gravity $M_{\text{Planck}} \to \infty$, which deforms the quaternionic geometry into a hyper-Kähler limit. Since any hyper-Kähler manifold can be coupled to supergravity in a quaternionic manifold, this limiting process must smoothly interpolate between both geometries, and its description requires care. It implies to simultaneously "zooming-in" with appropriate k factors in order to recover non-trivial hyper-Kähler geometries [8]. This procedure has been demonstrated for specific cases, involving the quaternionic quotient method [9,10].

Although, as pointed out previously, the Heisenberg algebra is uniquely realized at the quaternionic level as Heisenberg $\ltimes U(1)$, two distinct hyper-Kähler spaces exist with Bianchi II symmetry, realized either as Heisenberg $\ltimes U(1)$ (biaxial), or as strict Heisenberg (trixial) [11]. The former corresponds indeed to the infinite-M_{Planck} limit of the quaternionic space sharing its isometry and describing the string perturbative corrections to the hypermultiplet manifold, whereas nothing is known about the latter.

The purpose of the present note is to elaborate on the hyper-Kähler space with strict Heisenberg isometry. This raises a number of interesting questions, some of which stand beyond Heisenberg symmetry:

- Under which conditions a hyper-Kähler space of a prescribed isometry can give rise to a quaternionic ascendent with the same or less symmetry?
- Conversely, what is the general limiting procedure for reaching smoothly a non-trivial hyper-Kähler space from a given quaternionic one?

Examples are known, where a quaternionic space is constructed starting from a four-dimensional hyper-Kähler geometry via an eight-dimensional hyper-Kähler cone [12,13]. An interesting relationship can be further settled amongst quaternionic spaces with an isometry and hyper-Kähler spaces with a rotational symmetry, equipped with a hyper-holomorphic connection (i.e. whose curvature is $(1,1)$ with respect to all complex structures in the hyper-Kähler family). This was developed in [14,15] from a mathematical point of view, and in [16] in a more physical framework. We will here provide an alternative and systematic algebraic procedure for a direct uplift,

[1] In the literature, manifolds with holonomy contained in $Sp(2) \times Sp(2n)$ and non-zero Ricci curvature are actually called quaternion-Kähler. In the four-dimensional case of interest here, they are Weyl-self-dual Einstein spaces.