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Cosmological constant from a deformation of the Wheeler–DeWitt equation

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Abstract

In this paper, we consider the Wheeler–DeWitt equation modified by a deformation of the second quantized canonical commutation relations. Such modified commutation relations are induced by a Generalized Uncertainty Principle. Since the Wheeler–DeWitt equation can be related to a Sturm–Liouville problem where the associated eigenvalue can be interpreted as the cosmological constant, it is possible to explicitly relate such an eigenvalue to the deformation parameter of the corresponding Wheeler–DeWitt equation. The analysis is performed in a Mini-Superspace approach where the scale factor appears as the only degree of freedom. The deformation of the Wheeler–DeWitt equation gives rise to a Cosmological Constant even in absence of matter fields. As a Cosmological Constant cannot exist in absence of the matter fields in the undeformed Mini-Superspace approach, so the existence of a non-vanishing Cosmological Constant is a direct consequence of the deformation by the Generalized Uncertainty Principle. In fact, we are able to demonstrate that a non-vanishing Cosmological Constant exists even in the deformed flat space. We also discuss the consequences of this deformation on the big bang singularity.

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1. Introduction

It is expected that the geometry of space–time cannot be measured below a minimum length scale, which is usually taken to be the Planck scale [1,2]. At this scale it is likely that quantum fluctuations of the space–time itself come into play, breaking therefore its description as a smooth manifold [3,4]. For instance, in string theory, the minimum length scale is the string length itself. This means that, in perturbative string theory, it is not possible to probe the space–time below the string length scale [5–8]. The appearance of a minimum measurable length scale has also been studied in the context of loop quantum gravity [9–12], in noncommutative field theories [13,14] and also in black hole physics [15,16]. Even though the existence of such a minimum length scale is predicted from various different approaches, it is not consistent with the usual Heisenberg uncertainty principle, which states that the position of a particle can be measured with arbitrary precision, if its momentum is not measured. This means that there is no minimum measurable length scale compatible with the usual Heisenberg uncertainty principle. To accommodate this mismatch, we need to introduce a Generalized Uncertainty Principle (GUP) [17,18]. As the uncertainty principle is closely related to the Heisenberg algebra, the generalization of the usual Heisenberg uncertainty principle to GUP deforms the Heisenberg algebra [19,20]. This in turns modifies the coordinate representation of the momentum operator, and this new representation for the momentum operator produces correction terms for all quantum mechanical phenomena [21,22]. A more general deformation of GUP which incorporates the effect of double special relativity [23,24], has also been studied [25,26]. This deformed Heisenberg algebra also has terms proportional to linear powers of momentum. Motivated from GUP, the full four momentum of a field theory has also been modified, and the gauge theory corresponding to this deformation of field theory has been constructed [27–30]. However, it is also possible to deform the second quantized commutator between the fields in a similar way. This has been done for the Wheeler–DeWitt (WDW) equation [31–34]. This deformed WDW equation has also been used for analyzing quantum black holes[1] [37]. The third quantization of this deformed WDW equation has also been studied [38]. In this analysis, the deformation parameter was analyzed perturbatively.

Motivated by the deformation of Heisenberg algebra by linear terms in momentum [25,26], a similar deformation of the second quantized commutator has been studied [39]. It may be noted that in the deformation of the first quantized theories, the GUP parameter can be related to the existence of an intrinsic measurable length scale in space. Such a relation between a physical phenomena and GUP deformation has not been studied for the second quantized theories. A remarkable feature of the WDW equation deformed by GUP is to avoid singularities in space–time [39]. This is principally due to the introduction of a minimum limit to the field resolution. Therefore, it is quite obvious to try to extend this interesting feature to other contexts, for example, the cosmological constant. Indeed, it may be noted that the WDW equation is equivalent to a Sturm–Liouville problem and the related eigenvalue can be interpreted as a cosmological constant [40–44]. In this context the cosmological constant is a measure of the degeneracy of the only energy eigenvalue of the WDW equation without matter fields which obeys the following equation, $\mathcal{H}\Psi = E\Psi$, with $E = 0$. It is true that an exact solution has been found by Vilenkin in ordinary GR with a factor ordering equal to $q = -1$ [45]. However, except this special case, no other exact solutions have been found in this context. It is for this reason that one promising

[1] Other applications of GUP on quantum black holes can be found in Refs. [35,36].