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ScienceDirect
Nuclear Physics B 905 (2016) 132–187
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Fusion rules for the logarithmic $N = 1$ superconformal minimal models II: Including the Ramond sector

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Received 18 December 2015; received in revised form 9 February 2016; accepted 10 February 2016

Available online 18 February 2016

Editor: Hubert Saleur

Abstract

The Virasoro logarithmic minimal models were intensively studied by several groups over the last ten years with much attention paid to the fusion rules and the structures of the indecomposable representations that fusion generates. The analogous study of the fusion rules of the $N = 1$ superconformal logarithmic minimal models was initiated in [1] as a continuum counterpart to the lattice explorations of [2]. These works restricted fusion considerations to Neveu–Schwarz representations. Here, this is extended to include the Ramond sector. Technical advances that make this possible include a fermionic Verlinde formula applicable to logarithmic conformal field theories and a twisted version of the fusion algorithm of Nahm and Gaberdiel–Kausch. The results include the first construction and detailed analysis of logarithmic structures in the Ramond sector.

1. Introduction

The last ten years have seen significant advances in the study of the so-called logarithmic conformal field theories [3–5], making it clear that such theories are neither pathological nor intractable. Rather, it is now recognised [6–8] that logarithmic theories successfully model non-

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http://dx.doi.org/10.1016/j.nuclphysb.2016.02.010

local observables in statistical lattice models [9–13] and string theories with fermionic spacetime symmetries [14–17]. From a mathematical point of view, "logarithmic" means that the relevant category of modules over the vertex operator algebra is non-semisimple. More precisely, it means that the hamiltonian acts non-diagonalisably on the quantum state space. This leads to many subtle mathematical questions and the field of logarithmic vertex operator algebras is now being actively pursued by mathematicians, see [18–21] for example.

In [1], we instigated a detailed study of certain logarithmic conformal field theories with $N = 1$ supersymmetry. These are the $N = 1$ *logarithmic minimal models*, corresponding to the universal vertex operator algebras associated with the Neveu–Schwarz algebra. Some abstract consequences of combining supersymmetry with logarithmic structures had already been studied in [22,23] and a detailed discussion of the $N = 1$ triplet models may be found in [24]. Our motivation, however, is a recent lattice-theoretic study, reported in [2], in which certain fused loop models were conjectured to have the $N = 1$ logarithmic minimal models as their continuum scaling limits. We do not work with these loop models, instead preferring to study certain aspects of the $N = 1$ logarithmic minimal models directly using field- and representation-theoretic methods.

In particular, we study the fusion rules of certain $N = 1$ representations known as Kac modules. Originally introduced non-constructively [9] for Virasoro logarithmic minimal models to describe conjectured limiting partition functions for boundary sectors of (non-fused) loop models, candidates for Virasoro Kac modules were proposed in [25], for certain models, then confirmed and generalised in [26]. In [1], we introduced the $N = 1$ Kac modules, following these papers and [2], investigating them and their fusion rules in the Neveu–Schwarz sector. Here, we extend this investigation to include the Ramond sector, overcoming the significant technical difficulties that result from working with twisted representations.

The two main tools that we develop for this investigation are a fermionic analogue of the "standard" Verlinde formula of [27] and a twisted version of the Nahm–Gaberdiel–Kausch fusion algorithm [28,29]. The standard Verlinde formula is the centrepiece of the standard module formalism that is being developed to analyse the modular properties of logarithmic conformal field theories [17,26,30–35]. Combining this formalism with simple current technology [36,37], as was done for the rational Verlinde formula in [38], we arrive at a Verlinde formula that gives the (super)character of a fusion product involving both Neveu–Schwarz and Ramond modules. On the other hand, the Nahm–Gaberdiel–Kausch algorithm gives an algorithmic means of explicitly constructing fusion products and analysing the resulting structures. Originally applying only to untwisted modules, a twisted generalisation was first discussed in [39]. We simplify this discussion significantly and detail the practical implementation of the algorithm, necessary for explicit fusion calculations with Ramond modules.

We begin, in Section 2, with a thorough review of the representation theory of the $N = 1$ superconformal algebras, focusing on Verma modules and Fock spaces. As the Neveu–Schwarz sector was discussed in [1], and is anyway very similar to Virasoro representation theory, we concentrate here on the Ramond sector. In particular, we detail the unusual subsingular vector structures of the Verma modules and Fock spaces corresponding to the case where the conformal highest-weight h and central charge c satisfy $h = \frac{c}{24}$, referring to [40–42] for a more complete treatment. The section concludes with the definition of an $N = 1$ Kac module. These modules play a central role in what follows.

Section 3 introduces the characters and supercharacters of the Neveu–Schwarz and Ramond Fock spaces, these playing the role of the standard modules of the theory. The S-matrix (here, the kernel of an integral transform similar to the Fourier transform) is computed and the results