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Nuclear Physics B 905 (2016) 242–250

NUCLEAR PHYSICS B

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Yang–Baxter invariance of the Nappi–Witten model

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Received 23 December 2015; received in revised form 17 February 2016; accepted 17 February 2016

Available online 23 February 2016

Editor: Hubert Saleur

Abstract

We study Yang–Baxter deformations of the Nappi–Witten model with a prescription invented by Delduc, Magro and Vicedo. The deformations are specified by skew-symmetric classical r-matrices satisfying (modified) classical Yang–Baxter equations. We show that the sigma-model metric is invariant under arbitrary deformations (while the coefficient of B-field is changed) by utilizing the most general classical r-matrix. Furthermore, the coefficient of B-field is determined to be the original value from the requirement that the one-loop β-function should vanish. After all, the Nappi–Witten model is the unique conformal theory within the class of the Yang–Baxter deformations preserving the conformal invariance.

1. Introduction

The Yang–Baxter sigma-model description, which was originally proposed by Klimcik [1], is a systematic way to consider integrable deformations of 2D non-linear sigma models. According to this procedure, the deformations are specified by skew-symmetric classical r-matrices satisfying the modified classical Yang–Baxter equation (mCYBE). The original work [1] has been generalized to symmetric spaces [2] and the homogeneous CYBE [3].

Yang–Baxter deformations of the $AdS_5 \times S^5$ superstring can be studied with the mCYBE [4] and the CYBE [5]. For the former case, the metric and B-field are derived in [6] and the

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http://dx.doi.org/10.1016/j.nuclphysb.2016.02.017

full background has recently been studied in [7,8]. For the latter case, classical r-matrices are identified with solutions of type IIB supergravity including γ-deformations of S^5 [9,10] and gravity duals of non-commutative gauge theories [11,12], in a series of works [13–20] (for a short summary, see [21]).

Lately, Yang–Baxter deformations of 4D Minkowski spacetime have been studied [22,23]. In [22], classical r-matrices are identified with exactly-solvable string backgrounds such as Melvin backgrounds and pp-wave backgrounds. In [23], Yang–Baxter deformations of 4D Minkowski spacetime are discussed by using classical r-matrices associated with κ-deformations of the Poincaré algebra [24]. Then the resulting deformed geometries include T-duals of (A)dS_4 spaces[1] and a time-dependent pp-wave background. Furthermore, the Lax pair is presented for the general κ-deformations [23,26].

As a spin off from this progress, it would be interesting to study Yang–Baxter deformations of the Nappi–Witten model [27]. The target space of this model is given by a centrally extended 2D Poincaré group. Hence the Yang–Baxter deformed Nappi–Witten models can be regarded as toy models of the previous works [22,23], because the structure of the target space is much simpler than that of 4D Minkowski spacetime. This simplification makes it possible to study the most general Yang–Baxter deformation. As a matter of course, it is exceedingly complicated in general, hence such an analysis has not been done yet.

In this article, we investigate Yang–Baxter deformations of the Nappi–Witten model by following a prescription invented by Delduc, Magro and Vicedo [28]. We show that the sigma-model metric is invariant under the deformations (while the coefficient of B-field is changed) by utilizing the most general classical r-matrix. Furthermore, the coefficient of B-field is determined to be the original value from the requirement that the one-loop β-function should vanish. After all, the Nappi–Witten model is the unique conformal theory within the class of the Yang–Baxter deformations preserving the conformal invariance (i.e., Yang–Baxter invariance).

2. Nappi–Witten model

In this section, we shall give a concise review of the Nappi–Witten model [27].

The Nappi–Witten model is a Wess–Zumino–Witten (WZW) model whose target space is given by a centrally extend 2D Poincaré group. The associated extended Poincaré algebra $\mathfrak{g}$ is composed of two translations P_i $(i = 1, 2)$, a rotation J and the center T. The commutation relations of the generators are given by

$$[J, P_i] = \epsilon_{ij} P_j \,, \quad [P_i, P_j] = \epsilon_{ij} T \,, \quad [T, J] = [T, P_i] = 0 \,, \tag{2.1}$$

where ϵ_{ij} is an anti-symmetric tensor normalized as $\epsilon_{12} = 1$. It is convenient to introduce a notation of the generators with the group index I like

$$T_I = \left\{P_1, P_2, J, T\right\} \qquad (I = 1, 2, 3, 4) \,. \tag{2.2}$$

Let us introduce a group element represented by

$$g = \exp\left(a_1\, P_1 + a_2\, P_2\right) \exp\left(u\, J + v\, T\right) . \tag{2.3}$$

By using this group element g, the left-invariant current A can be evaluated as

[1] T-dual of dS_4 can be derived as a scaling limit of η-deformed AdS_5 as well [25].