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Leptonic Dirac CP violation predictions from residual discrete symmetries

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Abstract

Assuming that the observed pattern of 3-neutrino mixing is related to the existence of a (lepton) flavour symmetry, corresponding to a non-Abelian discrete symmetry group G_f, and that G_f is broken to specific residual symmetries G_e and G_ν of the charged lepton and neutrino mass terms, we derive sum rules for the cosine of the Dirac phase δ of the neutrino mixing matrix U. The residual symmetries considered are: i) $G_e = Z_2$ and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$; ii) $G_e = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$ and $G_\nu = Z_2$; iii) $G_e = Z_2$ and $G_\nu = Z_2$; iv) G_e is fully broken and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$; and v) $G_e = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$ and G_ν is fully broken. For given G_e and G_ν, the sum rules for $\cos\delta$ thus derived are exact, within the approach employed, and are valid, in particular, for any G_f containing G_e and G_ν as subgroups. We identify the cases when the value of $\cos\delta$ cannot be determined, or cannot be uniquely determined, without making additional assumptions on unconstrained parameters. In a large class of cases considered the value of $\cos\delta$ can be unambiguously predicted once the flavour symmetry G_f is fixed. We present predictions for $\cos\delta$ in these cases for the flavour symmetry groups $G_f = S_4$, A_4, T' and A_5, requiring that the measured values of the 3-neutrino mixing parameters $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$, taking into account their respective 3σ uncertainties, are successfully reproduced.

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1. Introduction

The discrete symmetry approach to understanding the observed pattern of 3-neutrino mixing (see, e.g., [1]), which is widely explored at present (see, e.g., [2–5]), leads to specific correlations between the values of at least some of the mixing angles of the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix U and, either to specific fixed trivial or maximal values of the CP violation (CPV) phases present in U (see, e.g., [6–10] and references quoted therein), or to a correlation between the values of the neutrino mixing angles and of the Dirac CPV phase of U [11–15].[2] As a consequence of this correlation the cosine of the Dirac CPV phase δ of the PMNS matrix U can be expressed in terms of the three neutrino mixing angles of U [11–14], i.e., one obtains a sum rule for $\cos\delta$. This sum rule depends on the underlying discrete symmetry used to derive the observed pattern of neutrino mixing and on the type of breaking of the symmetry necessary to reproduce the measured values of the neutrino mixing angles. It depends also on the assumed status of the CP symmetry before the breaking of the underlying discrete symmetry.

The approach of interest is based on the assumption of the existence at some energy scale of a (lepton) flavour symmetry corresponding to a non-Abelian discrete group G_f. Groups that have been considered in the literature include S_4, A_4, T', A_5, D_n (with $n = 10, 12$) and $\Delta(6n^2)$, to name several. The choice of these groups is related to the fact that they lead to values of the neutrino mixing angles, which can differ from the measured values at most by subleading perturbative corrections. For instance, the groups A_4, S_4 and T' are commonly utilised to generate tri-bimaximal (TBM) mixing [18]; the group S_4 can also be used to generate bimaximal (BM) mixing [19][3]; A_5 can be utilised to generate golden ratio type A (GRA) [21–23] mixing; and the groups D_{10} and D_{12} can lead to golden ratio type B (GRB) [24] and hexagonal (HG) [25] mixing.

The flavour symmetry group G_f can be broken, in general, to different symmetry subgroups G_e and G_ν of the charged lepton and neutrino mass terms, respectively. G_e and G_ν are usually called "residual symmetries" of the charged lepton and neutrino mass matrices. Given G_f, which is usually assumed to be discrete, typically there are more than one (but still a finite number of) possible residual symmetries G_e and G_ν. The subgroup G_e, in particular, can be trivial, i.e., G_f can be completely broken in the process of generation of the charged lepton mass term.

The residual symmetries can constrain the forms of the 3×3 unitary matrices U_e and U_ν, which diagonalise the charged lepton and neutrino mass matrices, and the product of which represents the PMNS matrix:

$$U = U_e^\dagger U_\nu \,. \tag{1}$$

Thus, by constraining the form of the matrices U_e and U_ν, the residual symmetries constrain also the form of the PMNS matrix U.

In general, there are two cases of residual symmetry G_ν for the neutrino Majorana mass term when a portion of G_f is left unbroken in the neutrino sector. They characterise two possible approaches — direct and semi-direct [4] — in making predictions for the neutrino mixing observables using discrete flavour symmetries: G_ν can either be a $Z_2 \times Z_2$ symmetry (which

[2] In the case of massive neutrinos being Majorana particles one can obtain under specific conditions also correlations between the values of the two Majorana CPV phases present in the neutrino mixing matrix [16] and of the three neutrino mixing angles and of the Dirac CPV phase [11,17].

[3] Bimaximal mixing can also be a consequence of the conservation of the lepton charge $L' = L_e - L_\mu - L_\tau$ (LC) [20], supplemented by a μ–τ symmetry.