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# Kählerian effective potentials for Chern–Simons-matter theories

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**Abstract**

In this paper, we will calculate the effective potential for a theory of multiple M2-branes. As the theory of multiple M2-branes can be described by a Chern–Simons-matter theory, this will be done by calculating the Kählerian effective potential for a Chern–Simons-matter theory. This calculation will be performed in $\mathcal{N} = 1$ superspace formalism. We will initially study an Abelian Chern–Simons-matter theory, and then generalize those results to the full non-Abelian Chern–Simons-matter theory. We will obtain explicit expressions for the superpropagators for this theory. These superpropagators will be used to calculate the one-loop effective potential.


## 1. Introduction

It is known that the action for multiple M2-branes should have $\mathcal{N} = 8$ supersymmetry. This is because the superconformal field theory describing multiple M2-branes is dual to the eleven di-

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mensional supergravity on $AdS_4 \times S^7$. Furthermore, we have $AdS_4 \times S^7 \sim [SO(2,3)/SO(1,3)] \times [SO(8)/SO(7)] \subset OSp(8|4)/[SO(1,3) \times SO(7)]$. Thus, the supergroup $OSp(8|4)$ can get realized as $\mathcal{N} = 8$ supersymmetry of the field theory dual to the eleven dimensional supergravity on $AdS_4 \times S^7$. Furthermore, the on-shell degrees of this theory are exhausted by bosons and physical fermions. So, the gauge sector of the theory for multiple M2-branes cannot have any on-shell degrees of freedom. These requirements are met by a theory called the BLG theory [1–5]. This theory is a Chern–Simons-matter theory. However, the gauge fields in this theory are valued in a Lie 3-algebra rather than a conventional Lie algebra. This theory describes two M2-branes, and it is not possible to use the BLG theory to describe more than two M2-branes. This is because there is only one known example of finite dimensional Lie 3-algebra, and this example describes two M2-branes. However, by complexifying the matter fields, the gauge sector of the BLG theory can be written as sum of two Chern–Simons theories, with levels $k$ and $-k$. The gauge fields of these Chern–Simons field theories are valued in regular Lie algebra, and the matter fields transform in the bifundamental representation.

It is possible to relax the requirement of manifest $\mathcal{N} = 8$ supersymmetry and generalize this approach to a Chern–Simons-matter theory with $\mathcal{N} = 6$ supersymmetry. This theory is called ABJM theory and it coincides with the BLG theory for the only known example of the Lie 3-algebra [6,7]. The gauge symmetry of this theory is represented by Chern–Simons theories with the gauge group $U_k(N) \times U_{-k}(N)$, where $k$ and $-k$ are Chern–Simons levels. It may be noted that even though this theory only has manifest $\mathcal{N} = 6$ supersymmetry, its supersymmetry is expected to be enhanced to the full $\mathcal{N} = 8$ supersymmetry for $k = 1$ or $k = 2$ [8]. This enhancement occurs due to the effects generated from monopole operators. As the ABJM theory has a gauge symmetry, we will need to fix a gauge to calculate the Kählerian effective potential. This can be done by adding the gauge fixing and ghosts terms to the original ABJM action. The gauge fixing of the ABJM theory and the BRST symmetry for this theory have been throughly studied [9–12].

In this paper, we will analyze the Chern–Simons-matter theories in $\mathcal{N} = 1$ superspace. It may be noted that even though this will only have manifest $\mathcal{N} = 1$ supersymmetry, the actual theory will have higher amount of supersymmetry. We will use $\mathcal{N} = 1$ superspace formalism since the Kählerian effective potentials is well understood in $\mathcal{N} = 1$ superspace formalism [13–19]. It may be noted that the ordinary Chern–Simons theory does not get renormalized, except for a finite one-loop shift [20,21]. The renormalization of Chern–Simons theory coupled to matter fields has also been studied [22]. The renormalization of supersymmetric Chern–Simons-matter theories has also been studied [23–25]. The matter fields exist in the fundamental representation of the gauge group. However, in a theory of multiple M2-branes, the matter fields exist in the bi-fundamental representation of the gauge group. It is possible to express the action of two M2-branes as matter-Chen-Simons theory where the gauge fields are valued in a Lie 3-algebra, and the one-loop renormalization such a theory has also been analyzed [26,27]. The scattering amplitudes in the ABJM theory have also been studied [28,29]. However, it is important to study the correction to the Kählerian effective potential. This is because we expect to understand the dynamics of M5-branes by studding the M2-branes ending on M5-branes [30–41]. This analysis is done using the superspace formalism. So, we will need to understand the one-loop corrections to the Kählerian effective potential, to understand the quantum behavior of this theory. Furthermore, certain symmetries can be broken in the theory of multiple M2-branes. In fact, the inclusion of a mass term breaks the conformal invariance of the ABJM theory [42–45]. So, we will need to compute the effective Kählerian effective potential for various deformations of the ABJM theory. Even though we only compute the Kählerian effective potential for the ordinary ABJM theory,