Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 902 (2016) 95–114

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

An alternative Hamiltonian formulation for the Pais–Uhlenbeck oscillator

Ivan Masterov

Laboratory of Mathematical Physics, Tomsk Polytechnic University, 634050 Tomsk, Lenin Ave. 30, Russian Federation

Received 21 September 2015; received in revised form 5 November 2015; accepted 12 November 2015

Available online 17 November 2015

Editor: Stephan Stieberger

Abstract

Ostrogradsky's method allows one to construct Hamiltonian formulation for a higher derivative system. An application of this approach to the Pais–Uhlenbeck oscillator yields the Hamiltonian which is unbounded from below. This leads to the ghost problem in quantum theory. In order to avoid this nasty feature, the technique previously developed in [7] is used to construct an alternative Hamiltonian formulation for the multidimensional Pais–Uhlenbeck oscillator of arbitrary even order with distinct frequencies of oscillation. This construction is also generalized to the case of an $\mathcal{N}=2$ supersymmetric Pais–Uhlenbeck oscillator.

1. Introduction

Higher derivative theories attract interest mostly due to their nice renormalization properties [1,2]. The method to construct Hamiltonian formulation for such systems has been proposed by Ostrogradsky [3]. In general, Hamiltonians obtained in such a way contain terms linear in momenta and are unbounded from below. This leads to the ghost problem on quantization [4,5]. The desire to cure this problem stimulates the investigation of the Pais–Uhlenbeck (PU) oscillator [4].

E-mail address: masterov@tpu.ru.

http://dx.doi.org/10.1016/j.nuclphysb.2015.11.011

After applying an appropriate canonical transformation [4,6], Ostrogradsky's Hamiltonian for the multidimensional PU oscillator of order $2n$ with distinct frequencies of oscillation ω_k, $k = 0, 1, .., n-1$, takes the form

$$H = \frac{1}{2}\sum_{k=0}^{n-1}(-1)^{k+1}(p_i^k p_i^k + \omega_k^2 x_i^k x_i^k). \tag{1}$$

When conventional quantization scheme is applied, the harmonic oscillators with negative overall factor bring about troubles with unbounded from below energy spectrum and, hence, with the absence of the ground state [4,5]. This motivates a search for an alternative Hamiltonian formulation and quantization procedure which lead to physically viable quantum theory [7–19]. So far the efforts have been focused mostly on the one-dimensional PU oscillator of the fourth order [7–16,19]. In particular, an elegant method to obtain an alternative canonical formalism with positive-definite Hamiltonian has been formulated in [7]. This alternative formulation has been realized in two steps. At the first stage, two functionally independent integrals of motion which are quadratic in variables have been used so as to write down an ansatz for the Hamiltonian of the fourth-order PU oscillator. The second step implies the derivation of an appropriate Poisson structure.

An attempt to generalize the results in [7] to the case of higher order PU oscillator has been made in [8]. However, this generalization exhibits some features which seem to contradict each other. On the one hand, the alternative Hamiltonian in [8] is not positive definite. In this sense it is not better than Ostrogradsky's one. On the other hand, it was claimed in [8] that the quantum theory of the PU oscillator constructed with the use of the alternative Hamiltonian is ghost free. As will be demonstrated below, the reason is that the alternative Hamiltonian together with the Poisson structure in [8] do not reproduce the equation of motion of the original PU oscillator.

One of the goals of the present paper is to use the technique introduced in [7] in order to obtain an alternative Hamiltonian formulation for the multidimensional PU oscillator of arbitrary even order with distinct frequencies of oscillation. We also explain which claims in [8] are incorrect.

Recently, in [20,21] an $\mathcal{N} = 2$ supersymmetric extension of the PU oscillator has been constructed. It has been shown that the invariance of the model under time translations implies unbounded from below spectrum. The Hamiltonian can be presented as the sum of decoupled $\mathcal{N} = 2$ supersymmetric harmonic oscillators with alternating sign. The corresponding quantum theory is characterized by the presence of negative-norm states and by the absence of the ground state. Our second concern in this paper is the construction of an alternative Hamiltonian for an $\mathcal{N} = 2$ supersymmetric PU oscillator which is achieved by generalizing the method in [7] to the $\mathcal{N} = 2$ supersymmetric case.

The paper is organized as follows. In the next section we apply the method previously developed in Ref. [7] to obtain an alternative Hamiltonian formulation for the PU oscillator of order $2n$. In Section 3, in the same manner we construct an alternative Hamiltonian formalism for an $\mathcal{N} = 2$ supersymmetric PU oscillator. We summarize our results and discuss possible further developments in the concluding Section 4. Some technical details are given in Appendix.

2. An alternative Hamiltonian formulation for the PU oscillator

The equation of motion of the multidimensional PU oscillator of order $2n$ can be written in the following form [4]