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Black hole entropy in the Chern–Simons-like theories of gravity and Lorentz-diffeomorphism Noether charge

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Abstract

In the first order formalism of gravity theories, there are some theories which are not Lorentz-diffeomorphism covariant. In the framework of such theories we cannot apply the method of conserved charge calculation used in Lorentz-diffeomorphism covariant theories. In this paper we firstly introduce the total variation of a quantity due to an infinitesimal Lorentz-diffeomorphism transformation. Secondly, in order to obtain the conserved charges of Lorentz-diffeomorphism non-covariant theories, we extend the Tachikawa method [1]. This extension includes not only Lorentz gauge transformation but also the diffeomorphism. We apply this method to the Chern–Simons-like theories of gravity (CSLTG) and obtain a general formula for the entropy of black holes in those theories. Finally, some examples on CSLTG are provided and the entropy of the BTZ black hole is calculated in the context of the examples.

1. Introduction

There is a class of gravitational theories in $(2+1)$-dimension (e.g. Topological massive gravity (TMG) [2], New massive gravity (NMG) [3], Minimal massive gravity (MMG) [4], Zewi-dreibein gravity (ZDG) [5], Generalized minimal massive gravity (GMMG) [6], etc.), called the Chern–Simons-like theories of gravity [7]. In this work, we try to obtain a general expression for the entropy of the black hole solutions in the context of CSLTG.

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In the metric formalism of gravity for the covariant theories defined by a Lagrangian n-form L, Wald has shown that the entropy of black holes is the Noether charge associated with the horizon-generating Killing vector field evaluated at the bifurcation surface [8]. Presence of the purely gravitational Chern–Simons terms and mixed gauge gravitational ones gives rise to a non-covariant theory of gravity in the metric formalism. Tachikawa extended the Wald approach to include non-covariant theories. Hence, regarding this extension one can calculate the black hole entropy as a Noether charge in the context of non-covariant theories as well [1]. Another way (apart of the Tachikawa method) to obtain the entropy of black holes in the context of such theories has been studied in the papers [9–14], by an appropriate way.

It is known that the Lagrangian of the CSLTG can be written in the first order formalism, in which the spin-connection is considered as an independent Lorentz vector valued one-form. Applying the Wald method to calculate the Noether charges in the first order formalism, one can find that the Noether charges are proportional to ξ, where ξ is a Killing vector field corresponding to the conserved charge. It is clear that ξ must be zero on the bifurcation surface when we calculate the entropy of black hole, because ξ is the horizon-generating Killing vector field which is zero on the bifurcation surface. It seems disappointing at the first glance because it appears that the entropy will be zero, but it is not true. Recently, it has been shown that in approaching to the bifurcation surface, the spin-connection diverges in a way that the spin-connection interior product in ξ remains finite ensuring that there is no problem [15]. Another way to deal with this problem was proposed in [16], in which the authors chose the Cauchy surface where the event horizon does not lie on bifurcation surface. To avoid any confusion and for extending the Wald approach to include Lorentz invariance in addition to diffeomorphism invariance, Jacobson and Mohd have introduced the so-called Lorentz–Lie derivative [15]. Lorentz–Lie derivative is a generalization of the Lie derivative, and it is covariant under the Lorentz-diffeomorphism transformations. The authors of [15] demand that the Lorentz–Lie derivative vanishes when ξ is a Killing vector filed.

It is clear that the CSLTG are manifestly diffeomorphism covariant theories but their Lagrangian may be non-invariant under the Lorentz gauge transformations. In this work we try to extend the Tachikawa method to be able to calculate Noether charges of a theory which is not covariant under the general Lorentz-diffeomorphism transformations.

We obtain a generic formula for the entropy of all stationary black hole solutions of any Chern–Simons-like theory of gravity, such as TMG, NMG, GMG, MMG, GMMG, ZDG, etc. It is interesting that our formula is very simple. One only need to know the coupling constants and the field content of the model, specially for the stationary black hole solution of such theories, where the horizon of black hole is a circle.

We use lower case Greek letters for the spacetime indices, and the internal Lorentz indices are denoted by lower case Latin letters. The metric signature is mostly plus.

2. Lorentz–Lie derivative and total variation

Suppose that the dimension of spacetime is n. Let $e^a{}_\mu$ denote the vielbein. Under a Lorentz gauge transformation, $e^a{}_\mu$ transforms as $\tilde{e}^a{}_\mu = \Lambda^a{}_b e^b{}_\mu$ where $\Lambda \in SO(n-1,1)$, i.e. $e^a = e^a{}_\mu dx^\mu$ is $SO(n-1,1)$ vector valued 1-form. The Lorentz–Lie derivative (L–L derivative) of e^a defined as follows [15]:

$$\mathfrak{L}_\xi e^a = \pounds_\xi e^a + \lambda^a{}_b e^b, \tag{1}$$