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Yang–Mills solutions and dyons on cylinders over coset spaces with Sasakian structure

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Abstract

We present solutions of the Yang–Mills equation on cylinders $\mathbb{R} \times G/H$ over coset spaces of odd dimension $2m + 1$ with Sasakian structure. The gauge potential is assumed to be $SU(m)$-equivariant, parameterized by two real, scalar-valued functions. Yang–Mills theory with torsion in this setup reduces to the Newtonian mechanics of a point particle moving in $\mathbb{R}^2$ under the influence of an inverted potential. We analyze the critical points of this potential and present an analytic as well as several numerical finite-action solutions. Apart from the Yang–Mills solutions that constitute $SU(m)$-equivariant instanton configurations, we construct periodic sphaleron solutions on $S^1 \times G/H$ and dyon solutions on $i\mathbb{R} \times G/H$.

1. Introduction

Higher-dimensional Super-Yang–Mills theory appears in the context of string theory for example as the low-energy limit of the heterotic superstring. In this limit, heterotic string theory yields ten-dimensional supergravity coupled to $\mathcal{N} = 1$ supersymmetric Yang–Mills theory [1,2]. In four dimensions, the full Yang–Mills equation is implied by the instanton equation, a first-order anti-self-duality equation. This fact generalizes to dimensions greater than four. The higher-dimensional instanton equation is particularly interesting for string compactifications on manifolds of the form $M_{10-d} \times X^d$ with compact part X^d and maximally symmetric flat space

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M_{10-d}. Requiring the gauge field on the compact manifold to satisfy the instanton equation is closely related to the preservation of $\mathcal{N} = 1$ supersymmetry on the non-compact spacetime part. Instantons in higher dimensions were first studied in [3], and solutions to the generalized anti-self-duality equation have been constructed, for example, in [4–10].

The requirement of supersymmetry preservation in string compactifications of the above type translates to a condition on the geometry of the compact internal manifold: imposing the instanton equation on the gauge field on X^d is equivalent to requiring reduced holonomy on the compact space. In heterotic compactifications, Calabi–Yau 3-folds have therefore been the preferred choice for compactification spaces, leading to phenomenologically interesting models with $\mathcal{N} = 1$ supersymmetry. Furthermore, G_2-holonomy 7-manifolds as well as 8-manifolds with $Spin(7)$-holonomy have been of interest in more general models. A problem of heterotic Calabi–Yau compactifications is that they come with a number of scalar fields with undetermined vacuum expectation value. Some of these moduli can be fixed by allowing for nonvanishing p-forms, so-called fluxes, to exist on the internal compact manifold. Flux compactifications do address the moduli problem but enlarge the number of possible string backgrounds significantly, leading to the string landscape problem. For a review of flux compactifications, see for example [11–13].

Nontrivial background fluxes on the internal compact manifold imply a backreaction on the geometry, relaxing the condition on the holonomy of the manifold. X^d is no longer required to have reduced holonomy but to admit a G-structure, i.e. a reduction of the tangent bundle structure group from $GL(d)$ to some subgroup $G \subset GL(d)$. If the manifolds admit a real Killing spinor [14], they are equipped with a connection with totally antisymmetric torsion. In the following, we will be interested in a connection whose torsion is determined up to a real scaling parameter κ. We will consider Sasakian manifolds as a special type of Killing spinor manifolds of dimension $2m + 1$ with structure group $G = SU(m)$. For a particular choice of metric, these manifolds are in addition Einstein. Sasaki–Einstein manifolds have been studied in the context of non-compact flux backgrounds as AdS/CFT duals of confining gauge theories or, more precisely, as type IIB AdS vacua that lead to dual $\mathcal{N} = 1$ Super Yang–Mills theories coupled to matter [11,15–17].

In this paper, we concentrate on cylinders $\mathbb{R} \times G/H$ over coset spaces with Sasakian structure. We start by repeating the basics of Yang–Mills equations, G-structure and in particular Sasakian manifolds in Chapter 2. We use an $SU(m)$-equivariant ansatz for the gauge connection, parameterized by two real scalar functions, to write out the Yang–Mills equation in components in Chapter 3. This leads to a system of two coupled second-order ordinary differential equations, reducing Yang–Mills theory with torsion to the Newtonian mechanics of a point particle moving in $\mathbb{R}^2$ under the influence of a potential. The shape of this potential depends on the torsion parameter κ. The instanton case is recovered for $\kappa = 1$. This case has been first studied in [18,19] and can also be found in [20]. We derive the corresponding particle action in Chapter 4 and discuss the critical points of zero energy. For a special value of κ, the second-order equations can be solved analytically and yield a *tanh*-kink-type solution, similar to solutions discussed in earlier works [21,22]. We construct further finite-action solutions numerically. Considering $S^1 \times G/H$ instead of $\mathbb{R} \times G/H$, we obtain periodic solutions, so-called sphalerons, which are discussed in section 4.2. Taking the product space $i\mathbb{R} \times G/H$ instead of $\mathbb{R} \times G/H$ leads to a sign flip in the potential. Solutions to this case are known as dyons and can be constructed numerically. We present some of them in section 4.3.