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A quantum bound-state description of black holes

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Abstract

A relativistic framework for the description of bound states consisting of a large number of quantum constituents is presented, and applied to black-hole interiors. At the parton level, the constituent distribution, number and energy density inside black holes are calculated, and gauge corrections are discussed. A simple scaling relation between the black-hole mass and constituent number is established.

1. Introduction

Systems that can be characterised by a dimensionless parameter $N \gg 1$ are of considerable experimental and theoretical significance. Prominent examples include interacting Bose–Einstein condensates and baryons in the quantum theory of SU(N)-chromodynamics. In the case of Bose–Einstein condensates, N simply counts the bosons that constitute the system. In quantum chromodynamics, colour neutrality of baryons implies that N can be identified with the number of valence quarks confined inside the baryons. The main amenity offered by large-N systems is a natural expansion parameter given by $1/N$. In quantum chromodynamics this expansion parameter has a diagrammatic interpretation as planar dominance, which has been exploited, for instance, in the $1/N$-expansion of heavy baryons [1,2].

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A generic feature of large-N systems is their non-perturbative character. Even if the elementary interactions between individual constituents are consistent with a weak-coupling regime, the large number of constituents can lead to strong collective effects experienced by any individual constituent amidst the others. This suggests a mean-field description, which is well understood in the non-relativistic domain, when the Hartree approximation can be applied. Large-N systems in the relativistic domain, however, are much less understood from a theoretical point of view. Attempts to describe these systems based on, for instance, the Dyson–Schwinger equations are usually too complicated to allow for a consistent approximation scheme.

The purpose of this article is to provide an analytical and quantitative framework for realising a mean-field description of large-N systems in the relativistic domain. In the approach presented here, the mean field is provided by a non-trivial vacuum structure causing in-medium modifications of the constituent dynamics that can be related to collective binding effects. At this level, the bound-state description is similar to the one developed by Shifman, Vainshtein and Zakharov for using quantum chromodynamics as a predictive theory of hadrons. Besides the celebrated quark–hadron duality, certain vacuum condensates (Lorentz- and gauge-invariant compositions of fields in the normal-ordering prescription) of quarks and gluons [3–5] are central concepts in their approach. These condensates parametrise the non-trivial vacuum structure of quantum chromodynamics and allow to represent hadron properties at sufficiently low energies to account for confinement.

In contrast, the approach presented here does not intend to model confinement effects. Rather, condensates are used as phenomenological bookkeeping devices to parametrise the mean field experienced by individual constituents in large-N systems. Our main objective is to construct a solid theoretical framework which makes good use of these phenomenological ideas. As will be shown in detail, this leads to a representation of relativistic quantum bound-states qualifying as large-N systems in terms of an auxiliary current. Such a representation is valid both for the asymptotic framework pertinent to the scattering matrix, as well as for the construction of kinematical states associated with large-N systems. Thus, with the aide of the auxiliary current, the corresponding bound states can be reduced in the sense of Nishijima and Lehmann, Symanzik and Zimmermann [6,7], as well as in the usual sense of absorption and emission processes. Obviously, this is an important prerequisite for calculating static and dynamical properties of these bound states.

As an application, following a recent proposal put forward in [8–11],[1] black holes will be considered as large-N systems at a quantitative level strictly following the logic of the general bound-state formalism developed in this article. The key idea is to model black holes as quantum bound-states of $N \gg 1$ constituents in Minkowski space–time. Here, constituents include all graviton polarisations, in particular scalar gravitons. In this application, Minkowski space–time is not considered as a specific background geometry, rather it has the status of a distinguished space–time. Of course, Schwarzschild space–times are non-perturbative deformations of Minkowski space–time, in the sense that arbitrary many couplings between gravitons and the associated energy–momentum tensor have to be considered [13] in order to reproduce this geometry. But the bound-state description suggested here goes beyond a purely perturbative reconstruction. From a geometrical point of view, the condensates represent non-perturbative deformations of Minkowski space–time. Furthermore, the description allows to construct observables sensitive to the constituent structure inside the black hole, such as the momentum-

[1] For Schwarzschild black holes in the context of matrix models see [12].