Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 902 (2016) 387–414

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Master integrals for the four-loop Sudakov form factor

Rutger H. Boels [a], Bernd A. Kniehl [a], Gang Yang [b,c]

[a] II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
[b] Institut für Physik, Humboldt Universität zu Berlin, IRIS-Gebäude, Zum Großen Windkanal 6, 12489 Berlin, Germany
[c] State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

Received 28 October 2015; accepted 13 November 2015

Available online 29 November 2015

Editor: Stephan Stieberger

Abstract

The light-like cusp anomalous dimension is a universal function in the analysis of infrared divergences. In maximally ($\mathcal{N} = 4$) supersymmetric Yang–Mills theory (SYM) in the planar limit, it is known, in principle, to all loop orders. The non-planar corrections are not known in any theory, with the first appearing at the four-loop order. The simplest quantity which contains this correction is the four-loop two-point form factor of the stress tensor multiplet. This form factor was largely obtained in integrand form in a previous work for $\mathcal{N} = 4$ SYM, up to a free parameter. In this work, a reduction of the appearing integrals obtained by solving integration-by-parts (IBP) identities using a modified version of `Reduze` is reported. The form factor is shown to be independent of the remaining parameter at integrand level due to an intricate pattern of cancellations after IBP reduction. Moreover, two of the integral topologies vanish after reduction. The appearing master integrals are cross-checked using independent algebraic-geometry techniques explored in the `Mint` package. The latter results provide the basis of master integrals applicable to generic form factors, including those in Quantum Chromodynamics. Discrepancies between explicitly solving the IBP relations and the MINT approach are highlighted. Remaining bottlenecks to completing the computation of the four-loop non-planar cusp anomalous dimension in $\mathcal{N} = 4$ SYM and beyond are identified.

E-mail address: kniehl@desy.de (B.A. Kniehl).

http://dx.doi.org/10.1016/j.nuclphysb.2015.11.016

1. Introduction

Maximally ($\mathcal{N}=4$) supersymmetric Yang–Mills theory (SYM) offers perhaps the best chance of truly solving an interacting four-dimensional quantum field theory. In addition, it is a proven stepping stone to pioneer new computational technology, as the large amount of supersymmetry renders many computations much simpler than their non-supersymmetric counterparts. In many cases, the resulting technology has transformed computational power in generic quantum field theories. For instance, in QCD, recent years have seen a boom of next-to-leading-order (NLO) computations considered unfeasible only a few years before, inspired ultimately by Witten's twistor string [1].

The focus of this paper is the computation of the *Sudakov* form factor, which is an observable that involves one gauge-invariant operator from the stress tensor multiplet and two on-shell massless states,

$$\mathcal{F}_2 = \langle p_1, p_2|\mathcal{O}|0\rangle. \tag{1}$$

In $\mathcal{N}=4$ SYM, this form factor was first discussed and computed at the two-loop order in [2]. It is noteworthy that the three-loop form factor was first computed in QCD [3], almost three years after the master integrals were found in [4]. This result was then adapted to provide the three-loop answer in $\mathcal{N}=4$ SYM in [5]. The latter computation cleanly shows that the computation within $\mathcal{N}=4$ SYM is technically much more straightforward, especially when employing modern unitarity-based methods instead of Feynman graph techniques.

Since there is only a single scale in the problem of computing the Sudakov form factor, the dependence on this scale is fixed by dimensional analysis. Hence, the form factor evaluates to a function which only depends on the dimensional regularization parameter ϵ ($D=4-2\epsilon$), the coupling constant and the number of colors N_c. Moreover, the dependence on ϵ is governed to a large extent by the known structure of infrared (IR) divergences. The divergent structure through $\frac{1}{\epsilon^2}$, for instance, is determined by the so-called cusp (or soft) anomalous dimension, which gets its name from its appearance in the computation of the light-like cusped Wilson line [6,7]. The cusp anomalous dimension is a function which is universal for a given quantum field theory.

In $\mathcal{N}=4$ SYM, an integral equation which determines the leading planar part of the cusp anomalous dimension in principle exactly was derived from integrability [8]. This is to date the most powerful and precise manifestation of the cluster of ideas known as the AdS/CFT correspondence [9]. However, much less is known about the non-planar part of the cusp anomalous dimension. At weak coupling, an immediate problem is that the first non-planar correction appears at four loops and has never been calculated to date, in any theory. Further motivation comes from a conjecture [10] that this correction may in fact vanish. This is based on the assumed completeness of the dipole contributions, which is consistent with the analysis of IR divergences in Yang–Mills theories explored through three-loop order, see also [11]. However, there is evidence from the Regge limit analysis in [12] that this naive dipole summation is not sufficient at four loops. It is important to settle this issue by explicit computation.

The integrand of the form factor was obtained in [13] using color-kinematic duality [14–16] as an ansatz generator. The coefficients in the ansatz were fixed by a choice of unitarity cuts, up to one remaining parameter. To evaluate the integral, a major next step is to perform the integral reduction, in particular using integration-by-parts (IBP) reduction [17,18]. This turns out